# SCRAMJET NOZZLE DESIGN USING METHOD OF CHARACTERISTICS

By

**T. Murugan**

DEPARTMENT OF AEROSPACE ENGINEERING

Indian Institute of Technology Kanpur

FEBRUARY, 2003

# SCRAMJET NOZZLE DESIGN USING METHOD OF CHARACTERISTICS

*A Thesis Submitted*

In Partial Fulfillment of the Requirements

for the Degree of

*Master of Technology*

by

T.Murugan

to the

**DEPARTMENT OF AEROSPACE ENGINEERING**

**INDIAN INSTITUTE OF TECHNOLOGY KANPUR**

*FEBRUARY, 2003*

POST GRADUATE OFFICE
THESIS SUBMITTED
ON 25-2-03
I.I.T., KANPUR 16

# CERTIFICATE

It is certified that the work contained in the thesis entitled "**SCRAMJET NOZZLE DESIGN USING METHOD OF CHARACTERISTICS**", by "Murugan", has been carried out under my supervision and that this work has not been submitted elsewhere for the award of a degree.

**(Prof. R. K. Sullerey)**
Thesis Supervisor
Department of Aerospace Engineering
I.I.T. Kanpur

Kanpur
February 2003

# ABSTRACT

Supersonic combustion Ramjets (Scramjets) are used for hypersonic flight speeds where the combustion takes place supersonically. For Scramjets, Literature shows that the Method of characteristics (MOC) is a good technique for designing a proper nozzle contour. A general 'C' language computer code is developed for designing a Scramjet nozzle contour using the Method of Characteristics numerical technique Initially a nozzle contour is designed for parallel flow at the exit. The effect of back pressure on nozzle exit at different altitude is evaluated. The variation of exit parameters and nozzle contour for different assumed values of constant-gamma and variable-gamma throughout the nozzle is studied for air. Variable-gamma is assumed as a function of temperature and chemical composition of the exhaust mixture. For different fuels, the species compositions of the exhaust mixture are calculated using stoichiometric equations. The species concentration is assumed constant throughout the length of the nozzle. The gamma variation after each characteristic is evaluated which gives the correct nozzle contour for Scramjet nozzle. The developed code can be used for both two-dimensional and axi-symmetric nozzles.

The results obtained from the MOC are verified with the available results in the literature. The factors that affect the nozzle performance are analyzed and calculated at different axial locations. The final nozzle contour is obtained by truncating the length at particular location where the performance parameters are optimum. From analysis, it is found that 60% of stream thrust function value gives a reasonable length. Force and moment coefficients along the nozzle contour at various stations are computed which is used for controlling the vehicle where the aerodynamic controls are ineffective. Flow through the designed nozzle is analyzed with an available Euler's code. The exit parameters obtained from the Euler's code are in good agreement with the MOC results.

*Dedicated*

*to*

*My Parents*

# ACKNOWLEDGEMENT

I feel great pleasure in expressing my sincere thanks to my guide Prof. R. K. Sullerey, who has been giving an excellent guidance, support and criticism throughout my thesis work I also thank Prof. E. Rathakrishnan for his valuable tips, which helped me a lot to gain basic knowledge in gas dynamics. I am thankful to him for extending his lab facilities to me like his own student.

I sincerely thank Mr. Thiyagarajan for his valuable suggestions and help during my M Tech thesis. I also thank Mr. Shibu Clement and Pradeep for their help during various stages of my work. I thank Mr. Siva Kumar for his encouragement and help at various stages of my M.Tech program.

I am extremely fortunate to have Mr. Kannan and Mr. Arun as my seniors. Their brotherly affection and cooperation made my Master Degree life peaceful and enjoyable at the early stage of my M. Tech program I also thank Kishore Kumar, Kapil Kumar and Govindarajan for their advice and timely help at various stages in my life. Their friendship made my B. Tech life is a memorable one in my life. We had lot of fun in MIT, which I will remember throughout my life. I also thank all my friends especially Ganesh, Athimoolam, Pugal, Muthu Kumaran, Lourdu Raj, Bala Murali, Bala Murugan, Jaya Kanthan and Siva Balan for their timely help.

Finally, I am so indebted to my parents who have sacrificed their own wishes and happiness for me. I cannot imagine my present position without their care, love, sacrifice, wishes, prayers and support.

# CONTENTS

# LIST OF FIGURES

# LIST OF TABLES

# NOMENCLATURE

## Alphabets

| | |
|---|---|
| A | Area |
| A, B, C | coefficients of partial differential equations |
| c | Local speed of sound |
| $C_F$ | Force coefficient |
| $C_M$ | Moment coefficient |
| $C_P$ | Total specific capacity |
| $C_{P\infty}$ | Free stream Specific Heat Capacity |
| D | Denominator |
| g | Gravitational acceleration |
| H | Height |
| h_i | Nozzle inlet height |
| i | Index |
| m | Total mass of a substance |
| M | Molecular weight of a substance |
| M | Mach number |
| $m$ | Mass flow rate |
| N | Numerator |
| $n$ | Number of moles |
| P | Static pressure |
| R | Universal gas constant |
| $Sa$ | Stream Thrust Function |
| T | Net thrust |
| t | Time |
| turn | Turning angle |
| V | Resultant velocity |
| u, v, w | Components of velocity along x, y, z respectively |

| | |
|---|---|
| x, y, z | Coordinate directions |
| Pres | Static pressure at a given altitude |
| Temp | Static temperature at a given altitude |
| Pres | Static density at a given altitude |

## Greek letters

| | |
|---|---|
| $\rho$ | Local density |
| $\phi$ | Potential function |
| $\upsilon$ | Prandtl-Mayer function |
| $\theta$ | Flow turning angle |
| $\alpha$ | Mass fraction |
| $\mu$ | Mach angle |
| $\gamma$ | Specific heat ratio |

## Subscripts

| | |
|---|---|
| * | Throat condition |
| i | Inlet condition |
| e | Exit condition |
| W | Wall condition |
| C | Cowl |

# CHAPTER 1

# INTRODUCTION

Propulsion is a science of designing an engine to propel a vehicle forward or upwards. In aviation, propulsion is classified into two types namely, air breathing and rocket propulsion. Both the types of propulsions, work on the same principle of pushing high velocity exhaust gases out at the back end, but are different in one important way. An air-breathing engine uses the atmospheric air as oxidizer, which makes it usable only within the atmosphere. A rocket engine is made for traveling in space. It would carry both fuel and oxidizer. So the specific impulse of the air-breathing engine is high compared to the rocket engine. Turbo jet engine has a specific impulse of around 3000 s, which is efficient up to Mach number 2.5. But rocket engines have only around 400 s. Turbo jet is useful for a particular case when high acceleration and high speed is not important, but most missile applications require high thrust and acceleration. Ramjets are attractive, in the high supersonic speed range, but initiation of this speeding up process requires a booster. If Mach number is more than 5, Scramjets (Supersonic combustion Ramjet) are preferred.

Ramjets are used in the flight Mach number ranging from 3 to 6. Both Ramjets and Scramjets need not be symmetric about the centre line of the engine, because they do not have any rotary mechanism. It is often convenient that the bottom surface of the vehicle itself is acting as an internal boundary of the engine. When the Mach number goes beyond 5, Scramjets are preferred because the losses in the Ramjets are quite large.

Scramjets are normally preferred for the flight Mach number range of 5-20. When the airflow is decreased by either Scramjet or Ramjet, both the relative velocity and kinetic energy of the incoming flow is reduced. This increases the internal energy of the flow coming into the combustor. For Mach numbers above 6, this effect is predominant. It is no longer advantageous to decelerate the flow to subsonic speed, as the inlet static

pressure increases to very high value This leads to the failure of burner structure, excessive wall heat transfer, dissociation of large fraction of available chemical energy, and pressure loss in the combustion chamber. So for hypersonic Mach numbers, Scramjets are preferred.

## 1.1 Nozzles

The function of the expansion system is to accelerate the flow from the burner exit static pressure to local atmospheric pressure over the entire range of vehicle operation in a controllable and reliable manner with maximum performance. This is accomplished by converting the thermal energy of the hot combustion chamber gases into kinetic energy. In simpler terms, nozzle is the component of a rocket or air-breathing engine that produces thrust.

## 1.2 Types of Nozzles

There are three major and basic types of nozzles used in aircraft applications namely conical nozzles, bell or contour nozzles and annular or plug nozzles. All other nozzles are modified from one of these basic types.

### 1.2.1 Conical nozzle

The conical nozzle was often used in early rocket applications because of its simplicity and ease of construction. The cone gets its name from the fact that the walls diverge at a constant angle. A small angle produces greater thrust, because it maximizes the axial component of exit velocity, but minimizing the angle makes the nozzle longer and heavier, which leads to complexities while building them. At the other extreme, size and weight are minimized by a large nozzle wall angle. Unfortunately, large angles reduce performance at low altitude, because the high ambient pressure causes overexpansion and flow separation. Here, the exit flow can be achieved with some angularity, which reduces the axial thrust of the vehicle.

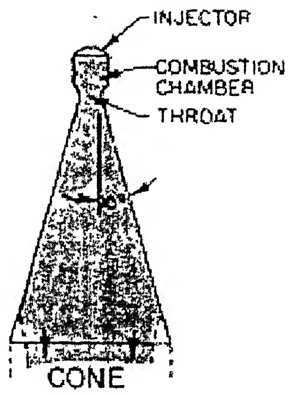


**Fig. 1.1 Conical Nozzle**

### 1.2.2 Bell Nozzle

This is the most commonly used nozzle shape, offering significant advantages over the conical nozzle, both in size and performance. Near the throat, the nozzle diverges at a relatively large angle but the degree of divergence tapers off further downstream of the nozzle. Near the nozzle exit, the divergence angle is very small. In this way it minimizes weight while maximizing performance. The most important design issue is to contour the nozzle to avoid oblique shocks and to maximize performance. However, the final bell shape will only be the optimum at a particular altitude.

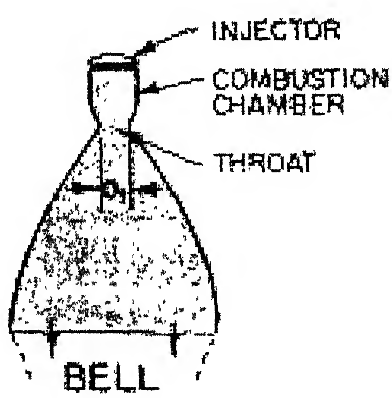


**Fig. 1.2 Bell Nozzle**

### 1.2.3 Single expansion ramp nozzle

Single expansion ramp nozzles are mainly used for hypersonic air breathing vehicles and missiles where the drag is an important performance parameter. In these nozzles, the bottom surface of the nozzle is removed because this portion produces large amount of drag compare to thrust. The expansion is obtained from simple sharp corner placed at the inlet of the nozzle. These nozzles are more efficient as compared to conical nozzles. Fig. 1.3 shows one such nozzle.

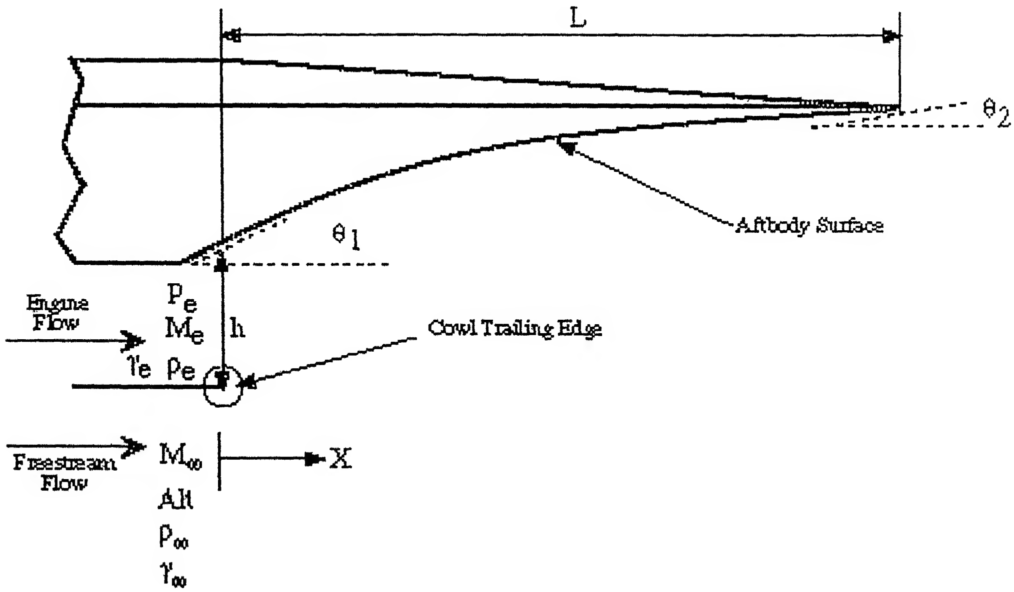


**Fig. 1.3 Single expansion ramp nozzle**

## 1.2.4 Annular Nozzles

The annular nozzle, also known, as the plug or "altitude-compensating" nozzle is the least employed of those discussed due to its greater complexity. The term "annular" refers to the fact that combustion occurs along a ring, or annulus, around the base of the nozzle "Plug" refers to the center body that blocks the flow from what would be the center portion of a traditional nozzle. "Altitude-compensating" is sometimes used to describe

these nozzles since that is their primary advantage. When considering an annular nozzle, the area of the center body ($A_{plug}$) must also be taken into account.

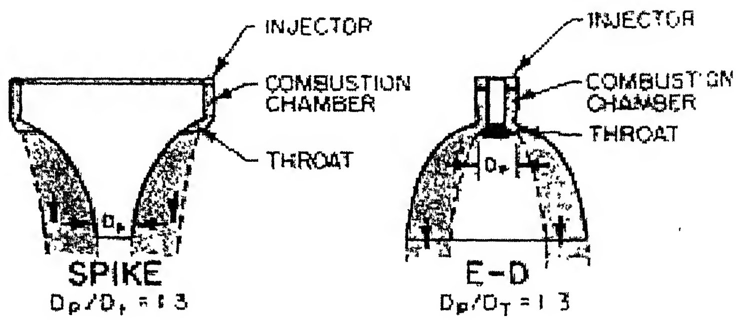


**Fig. 1.4 Plug Nozzle**

## 1.3 Application of Nozzles

Aero spike nozzle is mostly used for rocket applications. This nozzle is also called altitude compensation nozzle. Conical nozzle is also used for both rocket and hypersonic vehicles. Contour nozzle is mainly used for supersonic wind tunnels to get parallel flow in the test section. Recently they are being considered for hypersonic air breathing propulsion to increase the thrust. In order to design such a nozzle, initially a hypersonic parallel flow nozzle is designed using Method of characteristics numerical technique, and then this nozzle is truncated for rocket application, using various performance parameters. It is important to note that all conventional supersonic aircrafts have contour nozzle.

## 1.4 Nozzle Design

The function of the ideal nozzle is to provide parallel flow at the nozzle exit for any nozzle exit conditions. Any divergence in the nozzle leads to loss in its performance. Hence the contour nozzles are preferred for aircraft and wind tunnel applications In contour nozzle the exit uniform flow depends on exit Mach angle. If exit Mach number is greater, then the length of the nozzle also increases to very high value. In hypersonic vehicle the length of the nozzle is an important parameter. If the length increases then the

drag produced by this length itself is very large, so there will be more drag in addition to parallel flow and high thrust. The reduction in length is accomplished by using sharp corner expansion instead of continuous expansion. Method of characteristics is widely used for more practical nozzle design. This method is also used to design a nozzle for thermally perfect and chemically reacting flow with both equilibrium and non-equilibrium effects. This is a simple numerical method for designing a nozzle contour. In this method the flow is inviscid and irrotational.

Engineers in the conceptual air breathing vehicle design need to be able to assess engine performance at every point in a vehicle's trajectory. Unlike rocket engines, where propulsion system is dependent only on the vehicles altitude, air breathing propulsion system performance is strongly dependent on the flight path (velocity, dynamic pressure) of the vehicle as well.

In the rocket or air breathing missiles, the nozzle not only produces the thrust, but also produces a normal force on the nozzle surface of vehicle. This is an additional source of lift available on the vehicle. The pressure distribution along the nozzle surface greatly affects the pitching moment of the vehicle. This pitching moment is produced because of the expansion surface of the nozzle. These moments influence the stability of the vehicle. The force and the moment coefficients should be calculated precisely so that the equilibrium flight is sustained when the engine power is switched off. These moments are also used for adjusting the path of the missile. This phenomenon is called thrust vectoring. Here the hypersonic nozzle contour is designed and optimized for single expansion ramp nozzle, which is used in air breathing missile propulsion. Force and moment coefficient and the overall thrust of the vehicle are also calculated. The length of the contour nozzle is also reduced based on different parameters. Finally the flow angularity also calculated. Details of the analysis are given in the following chapters.

# CHAPTER 2

# LITERATURE SURVEY

Encountering propulsion problems are complex in real life, as interplay of heat and mass transfer, fluid mechanics, chemical kinetics, thermodynamics and turbulence occurs. Such problems are not tractable analytically, as the non-linear nature of governing partial differential equations involved in the flow makes it very difficult to get closed form solution. This leads to the existence of numerical methods. Method of characteristics is one such numerical technique, which is applicable for both super sonic and hypersonic nozzle design.

With an increasing demand for the reduced cost of space and missile activities, researchers are forced to investigate new technologies, such as increasing fuel economy. Greater fuel efficiency can be achieved in air breathing missiles and spacecrafts with the design of better propulsion and suitable selection of nozzle contours. In order to achieve maximum thrust of the propulsion system, recent researches focus on proper designing of the after body (nozzle). A push for increased technology in this field has lead to the development of optimum air breathing vehicle nozzle contours that can operate effectively and reliably at high altitudes.

## 2.1 Existing methods for Nozzle design

In the early years of rocket and missile nozzle design, conical nozzles were designed because methodology for designing efficient contour nozzles did not exist. Although they were necessary at that time, conical nozzles were an inadequate solution. Such nozzles resulted in performance losses, because they did not produce axially directed exhaust. The angularity in the conical nozzle leads to incorrect nozzle design for wind tunnel applications. In some cases, these nozzles were excessively heavy because of their length, which leads to enormous drag. This is one of the main reasons for hypersonic flights were

not possible in early days. Therefore, the design of efficient nozzle and parallel flow exhaust was a challenging task. One of the major problems in designing a supersonic wind tunnel is the determination of appropriate nozzle contour. Eradicating this problem would yield a parallel and uniform flow in the test section.

Short and efficient contour nozzles were finally developed when the truncated perfect nozzle approach was pioneered. In this approach, a wind tunnel nozzle was designed for uniform flow and then truncated at a much shorter length. Using this approach, Ahlberg et al [1] presented some results that were obtained for supersonic nozzle. Hoffman [2] discussed some variation of the original method. With in the same period, an alternate approach to this problem, based on the calculus of variations, was formulated. This method was improved by Rao [3]. According to the procedure outlined by Rao, the length, ambient pressure, and flow properties in the immediate vicinity of the throat, are the governing conditions under which thrust is maximized.

This classical process assumes that an isentropic flow exists in the nozzle. The nozzle contour is constructed using Method of characteristics to give the desired flow at the exit. Historically, Rao's method has been considered the best, as the other classical approaches fail to surpass its performance. However, the drawback in Rao's method is its dependence on an inviscid and calorifically perfect gas. Classical procedures are based on an approximation of viscous effects. These methods rely on an inviscid design. Therefore, this method is inapplicable for high temperature gas flow in the hypersonic nozzle design and certain correction must be made for types of fuels used in the aircraft propulsion devices.

Formulae required for calculating the species specific heat capacity and specific heat ratio at very high temperature were proposed by Oussama Rizkalla [4]. Computational studies on calculated chemical and vibrational Non-equilibrium Effects in hypersonic flows were also done. In order to study vibrational effects of both equilibrium and Non-equilibrium conditions in hypersonic flow, some results from quantum mechanics were used. Calculations for both force and moment coefficients for axial and normal directions were

also made for a Mach number 20 flight condition. From these analyses, it was concluded that the vibrational relaxation time appear to be quite short at these conditions. Hence, the flow appeared to be in *vibrational equilibrium* throughout its passage through the nozzle.

The vehicle trim requirements were also determined from calculations that include both finite rate chemistry and viscous effects. Beyond a certain axial location, which is considerably near the upstream of the nozzle exit station, certain amount of additional axial thrust was obtained. This additional thrust is because of the added expansion surface in the after body of the nozzle. However, the added expansion surface significantly affects the normal force coefficient and its associated pitching moment. The pitching moment will significantly affect the vehicle trim requirements. Therefore, care should be taken while calculating the normal force and pitching moment.

A simple method for calculating the properties of combustion products was given by Guha [5]. The method is generic: the same equations apply to any fuel for a specific temperature range. This offers best accuracy comparable with the most elaborate method available but at a significantly reduced computational cost. The numerical methods presented here are for lean mixtures (excess air). For rich mixtures, when not all the fuel is burned, the chemistry of the combustion process can be complex. However, if the composition of the products of combustion is known, then this simple methodology can be extended to rich mixtures.

The MOC method would offer considerable savings in computational time with no significant loss of accuracy. So, this method is used to calculate the specific heat capacity of any combustion species once the temperature in the combustion chamber is known. In the current nozzle design, it is assumed that complete combustion takes place in the combustion chamber. The maximum combustion temperature is attained when hydrocarbon fuels are mixed with just enough air so that complete conversion of the hydrocarbons to water vapor and carbon dioxide occurs.

"An Introduction to Numerical Techniques for Nonlinear Supersonic Flow" was given by John D. Anderson [6], where both Method of characteristics and finite difference numerical methods were discussed. Meanwhile, newer methods to solve the nonlinear partial differential equations that exist in the potential flow were given by Michel Saad [7] These nonlinear partial differential equations were solved using linearized potential flow (small perturbation theory) and Method of characteristics technique. A detailed procedure of Method of characteristics design was also discussed.

A simple procedure for designing a low supersonic contour nozzle design was described in "Gas Dynamics" by E. RathaKrishnan [8] where the gas is assumed calorifically perfect. However, this cannot be used for supersonic and hypersonic nozzle design. Here, the present design is for thermally perfect gas where the chemical composition of the exhaust was frozen.

## 2.2 Scope of the work

Method of characteristics numerical method is used here for designing a hypersonic Single expansion ramp nozzle. The minimum length hypersonic contour nozzle for wind tunnel application is designed and the length of the nozzle is optimized for missile application using "C" language computer code. The nozzle contour is designed for different Mach number and altitude conditions. The length of the nozzle is minimized using different design parameters such as stream thrust function, force and moment coefficients.

Here, the combustion is assumed to be complete. So, using stoichiometric equation the exact species composition is found. From the given inlet, temperature in the nozzle, specific heat capacity and specific heat ratio are found using Guha's generic method [5]. The chemical composition of the nozzle exit is constant and the temperatures at different axial location are evaluated from simple isentropic relations. After this, nozzle contour is designed using Method of characteristics numerical method. Provisions are also provided for different altitude atmospheric condition calculations. These formulae are taken from

NASA Online tutorial [27]. This design is correctly expanded nozzle design. Force and moment coefficients are also calculated along the length of the nozzle using the formulae given by Oussama Rizkalla [4].

Stream thrust function from the nozzle inlet to exit is calculated. Stream thrust function is a non-dimensional function that is used to find the mass flow rate specific thrust of the vehicle. Momentum and pressure thrust along the nozzle is also calculated. Axial and normal force and moment coefficients are computed for entire length of the nozzle. The length of the hypersonic nozzle is optimized from the above-mentioned parameters. The normal force coefficient leads to additional lift on the nozzle section, which produces the pitching moment. The knowledge of pitching moment is useful for controlling the vehicle where the aerodynamic controls are ineffective.

The results obtained from this design is verified with Ref [4] available data and verified with Euler code where the flow is inviscid. In Euler code, the nozzle geometry is designed using, the results obtained from present work and the results obtained from Euler code are compared with MOC results.

# CHAPTER 3

# THEORY

Analytical solutions of the equations describing fluid flow have been found only for simplified problems. No general analytical solution has been found yet for non linear partial differential equations. For this reason numerical techniques have been used for solving non linear problems. One such technique is the Method of characteristics (MOC), used for ordinary two dimensional exhaust nozzles, which form the basis for design of Ramjet and Scramjet exhaust systems.

The MOC method can be applied to one dimensional unsteady flow, two and three dimensional steady flows and axi-symmetric steady flow .In all the cases flow must be irrotational and isentropic. The computer code developed here can be used for both two dimensional and axi-symmetric contour nozzles design.

The Method of characteristics indicates the properties of a perfect gas in which continuous waves of small but finite amplitude are present as the gas flow supersonically, irrotationally, and shock free in any passage.

From the equation of law of conservation of mass for unsteady flow,

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0 \qquad \text{.................................} \quad (1.1)$$

For steady (independent of time) flow, we can neglect the first term,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}+\frac{1}{\rho}\left(u\frac{\partial \rho}{\partial x}+v\frac{\partial \rho}{\partial y}+w\frac{\partial \rho}{\partial z}\right)=0 \qquad \text{.........................} \quad (1.2)$$

These components can be expressed in terms of the velocity potential as:

$$u = \frac{\partial \phi}{\partial x} = \phi_x$$

$$v = \frac{\partial \phi}{\partial y} = \phi_y$$

$$w = \frac{\partial \phi}{\partial z} = \phi_z$$

The continuity equation in terms of velocity potential may then be written as:

$$\phi_{xx} + \phi_{yy} + \phi_{zz} + \frac{1}{\rho}\left( \phi_x \frac{\partial \rho}{\partial x} + \phi_y \frac{\partial \rho}{\partial y} + \phi_z \frac{\partial \rho}{\partial z} \right) = 0 \qquad \text{.......................... (1.3)}$$

From the momentum equation, assuming that only pressure forces are present (neglecting Body force and viscous force),

$$\frac{dP}{\rho} + \frac{dV^2}{2} = 0 \qquad \text{.... ................................ (1.4)}$$

But $V^2 = u^2 + v^2 + w^2 = \phi^2{}_x + \phi^2{}_y + \phi^2{}_z$

Substituting this value in the above equation, we get

$$\frac{dP}{\rho} + \frac{1}{2}\left(\phi^2{}_x + \phi^2{}_y + \phi^2{}_z\right) = 0 \qquad \text{....... .................... ...... (1.5)}$$

We know that, for isentropic flow:

$$c^2 = \frac{dp}{d\rho}$$

Velocity of sound (c) is applicable for any flow (thermally perfect and chemically reacting flow).

Eliminating $dp$ from equation (5):

$$d\rho = -\frac{\rho}{2c^2} d\left(\phi^2{}_x + \phi^2{}_y + \phi_z{}^2\right) \qquad \text{.................................. .. (1.6)}$$

Partial differentiation of this equation with respect to the three coordinate axes gives:

$$\frac{d\rho}{dx} = -\frac{\rho}{c^2}\left(\phi_x \phi_{xx} + \phi_y \phi_{xy} + \phi_z \phi_{xz}\right)$$

$$\frac{d\rho}{dy} = -\frac{\rho}{c^2}\left(\phi_x \phi_{xy} + \phi_y \phi_{yy} + \phi_z \phi_{yz}\right)$$

$$\frac{d\rho}{dz} = -\frac{\rho}{c^2}\left(\phi_x \phi_{zx} + \phi_y \phi_{zy} + \phi_z \phi_{zz}\right)$$

Substituting these values in equation (3),

$$\phi_{xx} + \phi_{yy} + \phi_{zz} - \frac{1}{c^2}\begin{bmatrix} \phi_x\left(\phi_x \phi_{xx} + \phi_y \phi_{xy} + \phi_z \phi_{xz}\right) \\ + \phi_y\left(\phi_x \phi_{xy} + \phi_y \phi_{yy} + \phi_z \phi_{yz}\right) \\ + \phi_z\left(\phi_x \phi_{zx} + \phi_y \phi_{xy} + \phi_z \phi_{xz}\right) \end{bmatrix} = 0$$

$$\phi_{xx} + \phi_{yy} + \phi_{zz} - \frac{1}{c^2}\left(\phi^2{}_x \phi_{xx} + \phi^2{}_y \phi_{yy} + \phi^2{}_z \phi_{zz}\right) - \frac{2}{c^2}\left(\phi_x \phi_y \phi_{xy} + \phi_y \phi_z \phi_{yz} + \phi_z \phi_x \phi_{zx}\right) = 0 \qquad \text{.....................} (1.7)$$

This is the differential form of potential function in three dimensions.

For two dimensional flow,

$$\phi_{xx} + \phi_{yy} - \frac{1}{c^2}\left(\phi^2{}_x \phi_{xx} + \phi^2{}_y \phi_{yy}\right) - \frac{2}{c^2}\left(\phi_x \phi_y \phi_{xy}\right) = 0 \qquad \text{..........................} (1.8)$$

Rearranging this equation, we get

$$\left(1 - \frac{\phi^2{}_x}{c^2}\right)\phi_{xx} - \frac{2}{c^2}\left(\phi_x \phi_y \phi_{xy}\right) + \left(1 - \frac{\phi^2{}_y}{c^2}\right)\phi_{yy} = 0 \qquad \text{..........................} (1.9)$$

This equation is similar to second order partial differential equation with two independent variables,

$$A\frac{\partial^2\phi}{\partial x^2} + 2B\frac{\partial^2\phi}{\partial xy} + C\frac{\partial^2\phi}{\partial y^2} = 0 \qquad \text{.......................................... (1.10)}$$

Where,

$$A = 1 - \frac{\phi^2_x}{c^2}$$

$$B = -\frac{1}{c^2}\left(\phi_x\phi_y\right)$$

$$C = 1 - \frac{\phi^2_y}{c^2}$$

The terms A, B and C are functions of both the independent variables x, y and the dependent variables u, v. An important point to be noted here is that the velocity of sound c is not a variable. This can be expressed in terms of dependent variables.

There are many boundary value problems which involve partial differential equations Only few of these equations can be solved by analytical methods. In most of the cases we depend on the numerical solution of such partial differential equations. Of the various numerical methods available for solving these equations, the method of finite difference is commonly used. Computational fluid dynamics is one of such method used for aerospace applications.

Computational Fluid Dynamics (CFD) is a new third dimension in aerodynamics, complementing the previous dimensions of both pure experimental and pure theory. Basically, CFD is the art of replacing the governing partial differential equations of fluid flow with numbers, and advancing these numbers in space and time to obtain a final numerical description of the complete flow field of interest.

In this method, the derivatives appearing in the equation and the boundary conditions are replaced by their finite difference approximations. Then the given equation is changed to a difference equation which is solved by iterative procedure. This is slow but gives good results of boundary value problems. An added advantage of this method is that the computation can be done by electronic computer.

MOC is simple and efficient method for inviscid supersonic and hypersonic nozzle design over computational fluid dynamics. This method can be applied to one dimensional unsteady flow, two and three dimensional steady flows and axi-symmetric steady flow.

Equation (10) has three possible solutions,

(i) $B^2 - AC < 0$ Elliptic equation.

This equation has two imaginary roots. There is no specific direction for propagation of waves, so the wave will propagate like a spherical wave.

(ii) $B^2 - AC = 0$ Parabolic equation.

This equation has two identical real roots. The wave will propagate in only one direction.

(iii) $B^2 - AC > 0$ Parabolic equation.

This equation has two real and distinct roots. The wave will propagate in two distinct directions.

Now consider the equation (10),

$$B^2 - AC = \frac{V^2}{c^2} - 1$$

For Subsonic flow V<c, then $B^2 - AC < 0$, which is nothing but the elliptic equation.

For Sonic flow V=c, then $B^2 - AC = 0$, which is parabolic equation.

For Supersonic flow V>c then $B^2 - AC > 0$, which is hyperbolic equation.

The problem under consideration is Hypersonic nozzle design, where V>c.

Hence there are two distinct real roots. These solutions are associated with two families of characteristic curves in the x-y plane, which are known as physical characteristics.

As an example, consider a uniform isentropic supersonic flow past a convex corner shown below,

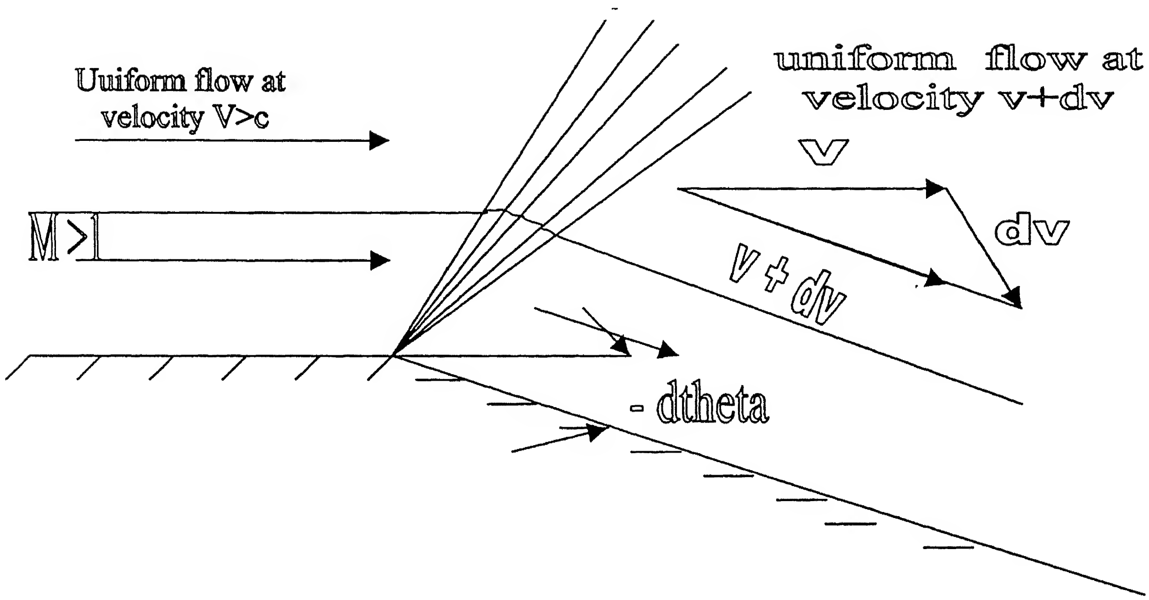


**Fig. 3.1 Flow over convex corner**

The Mach waves are generated at the corner, so that the gas flows uniformly, but at different velocity, while crossing each wave. Since the flow is shock free, the velocity is continuous, so that the first derivatives of the velocity potential $\frac{\partial \phi}{\partial x}$ and $\frac{\partial \phi}{\partial y}$ are continuous The gas expands as it flows around the corner, therefore increasing the velocity when the gas reaches the region downstream of the wave. This means that there is a discontinuity in the velocity gradient at the Mach wave.

Since the velocity components are continuous, the changes in velocity components are given as:

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy$$

*and* ........................................... (1.11)

$$dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy$$

However, the condition of irrotationality implies that $\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0$

From this

$$dv = \frac{\partial u}{\partial x}dx + \frac{\partial v}{\partial y}dy$$

Equation (10) can be written as,

$$A\left(\frac{\partial u}{\partial x}\right) + 2B\left(\frac{\partial u}{\partial y}\right) + C\left(\frac{\partial v}{\partial y}\right) = 0$$

Rearrange the Equation (11),

$$dx\left(\frac{\partial u}{\partial x}\right) + dy\left(\frac{\partial u}{\partial y}\right) + 0\left(\frac{\partial v}{\partial y}\right) = du$$

*and*

$$0\left(\frac{\partial u}{\partial x}\right) + dy\left(\frac{\partial u}{\partial y}\right) + dy\left(\frac{\partial v}{\partial y}\right) = dv$$

This set of equation can be solved for each of the three unknown.

These equations are solved using Cramer's rule,

$$\frac{\partial^2 \phi}{\partial x \partial y} = \frac{\begin{vmatrix} 1-\frac{u^2}{c^2} & 0 & 1-\frac{v^2}{c^2} \\ dx & du & 0 \\ 0 & dv & dy \end{vmatrix}}{\begin{vmatrix} 1-\frac{u^2}{c^2} & \frac{-2uv}{c^2} & 1-\frac{v^2}{c^2} \\ dx & dy & 0 \\ 0 & dx & dy \end{vmatrix}} = \frac{N}{D} \qquad \text{.............................. (1 12)}$$

Consider the fig below,

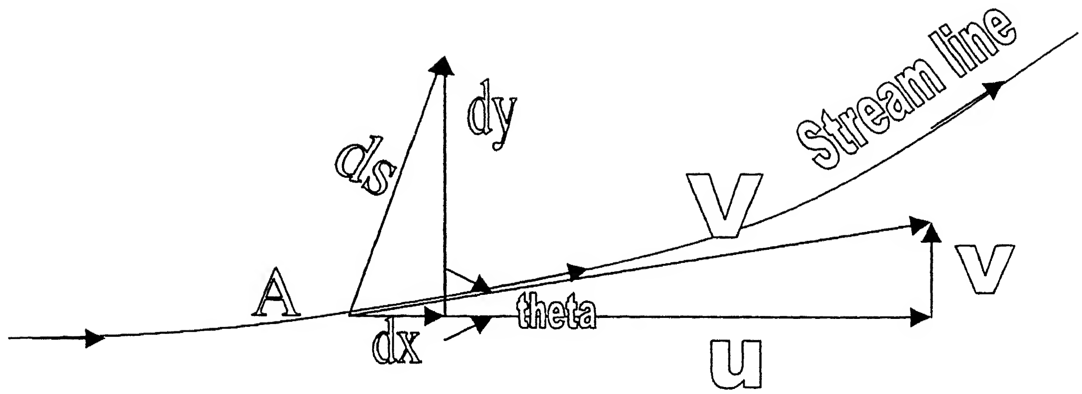


**Fig. 3.2 Flow description**

The derivative $\frac{\partial^2 \phi}{\partial x \partial y}$ has a specific value at point A. Equation (12) gives the value solution for $\frac{\partial^2 \phi}{\partial x \partial y}$ for an arbitrary choice of dx and dy. The combination of dx and dy define arbitrary direction ds away from point A. This direction is different from the stream line direction going through point A. The differential du and dv represent the change in velocity that takes place due to increments in dx and dy. Hence, although the choice of dx and dy is arbitrary, the value of du and dv must correspond to this choice No matter the value of dx and dy are arbitrarily chosen, the corresponding value of du and dv will always ensure obtaining the same value of $\frac{\partial^2 \phi}{\partial x \partial y}$ at point A.

The single exception to the above comments occurs when dx and dy are chosen so that D=0 in equation (12). In this case $\frac{\partial^2 \phi}{\partial x \partial y}$ is not defined. This situation will occur for a specific direction ds away from point A, defined for that specific combination of dx and dy for which D=0.

$$\frac{\partial^2 \phi}{\partial x \partial y} = \frac{N}{D} = \frac{0}{0}$$

Here, $\frac{\partial^2 \phi}{\partial x \partial y}$ is an indeterminate form. The important conclusion here is that there is some direction (or directions) through point A along which $\frac{\partial^2 \phi}{\partial x \partial y}$ is indeterminate. This implies that the derivatives of the flow variables are indeterminate along these lines. Hence, we have proved that lines do exist in the flow field along which derivatives of the flow variables are indeterminate .These lines are called characteristics lines.

Setting the numerator equal to zero, gives:

$$A\,(d\,y)^2 - 2B\;d\,x\;d\,y + C\,(d\,x)^2 = 0$$

$$\left(1-\frac{u^2}{c^2}\right)(d\,y)^2 + \frac{2uv}{c^2}\,d\,x\,d\,y + \left(1-\frac{v^2}{c^2}\right)(d\,x)^2 = 0$$

$$\left(1-\frac{u^2}{c^2}\right)\left(\frac{d\,y}{d\,x}\right)_{char}^2 + \frac{2uv}{c^2}\left(\frac{d\,y}{d\,x}\right)_{char} + \left(1-\frac{v^2}{c^2}\right) = 0 \qquad \text{.............. (1.13)}$$

$$\left(\frac{d\,y}{d\,x}\right)_{char} = \frac{-2uv/c^2 \pm \sqrt{\left(-2uv/c^2\right)^2 - 4\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}}{2\left(1-\frac{u^2}{c^2}\right)} \qquad \text{............ .... (1.14)}$$

$$\left(\frac{d\,y}{d\,x}\right)_{char} = \frac{uv/c^2 \pm \sqrt{\left(u^2+v^2\right)/c^2 - 1}}{1-u^2/c^2} \qquad \text{........................ (1.15)}$$

From above figure, the velocity components u and v are:

$$u = V\cos\theta$$
$$v = V\sin\theta$$

Substituting these values in equation (14), we get

$$\left(\frac{dy}{dx}\right)_{char} = \tan(\theta \mp \mu) \qquad \text{..........................................} \quad (1.16)$$

It states that *two* characteristics lines run through point A, one line with the slope equal to $\tan(\theta - \mu)$ and other with the slope equal to $\tan(\theta + \mu)$. These lines are shown below.

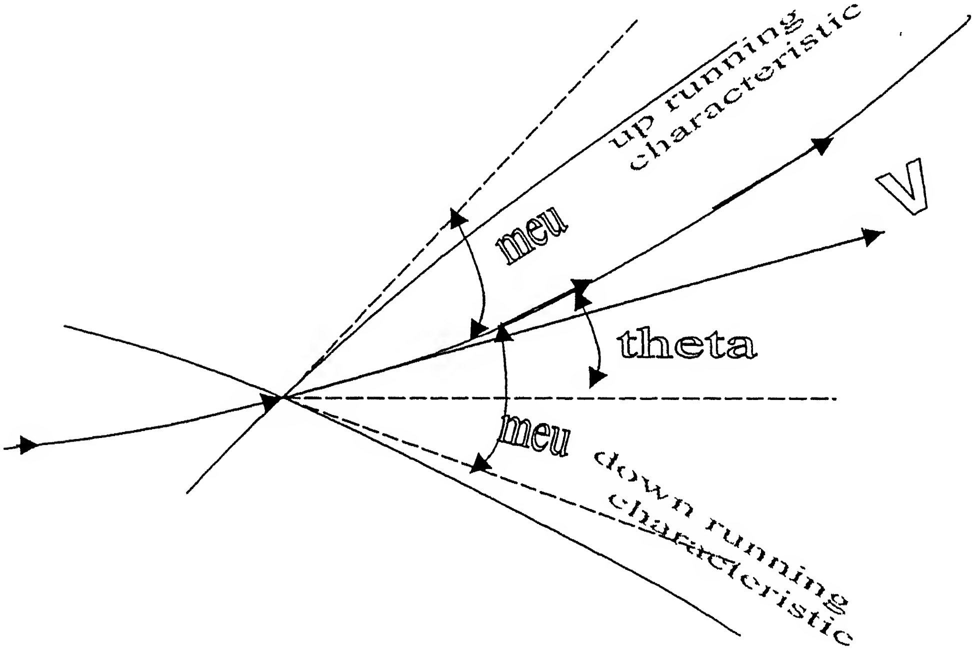


**Fig. 3.3 Inclination of characteristic curves**

The characteristic lines through point A are simply the right and left running Mach waves. Therefore, Each and every point the flow field has two characteristic lines, up and down running characteristics. It is very difficult to consider infinite number of characteristics in flow field analysis. For practical applications we deal with a finite number of waves. Note that the characteristic lines are curved in space because,

(1) The local Mach angle depends on the local Mach number, which is a function of x

and y

(2) The local free stream direction θ varies throughout the flow.

Now considering N=0, then

$$\left(1-\frac{u^2}{c^2}\right)du\,dy+\left(1-\frac{v^2}{c^2}\right)dx\,dv=0$$

$$\frac{dv}{du}=\frac{-\left(1-u^2/c^2\right)}{\left(1-v^2/c^2\right)}\left(\frac{dy}{dx}\right)_{char}$$

Substituting the value of $\left(\frac{dy}{dx}\right)_{char}$ from equation (14), we get

$$\frac{dv}{du}=\frac{uv\big/c^2\mp\sqrt{\left(u^2+v^2\right)\big/c^2-1}}{1-v^2/c^2} \qquad \text{.......................................... (1.17)}$$

Substituting the value of u and v in term of resultant velocity (V), and after some algebraic manipulation, we get:

$$d\theta=\mp\sqrt{M^2-1}\,\frac{dV}{V} \qquad \text{.......................................... (1.18)}$$

From energy equation,

$$\frac{c^2}{\upsilon-1}+\frac{V^2}{2}=Cons\tan t$$

But c = V/M, and there fore:

$$\frac{V^2}{M^2\left(\upsilon-1\right)}+\frac{V^2}{2}=Cons\tan t$$

Differentiation of this equation with respect to V leads to an expression for velocity change as a function of Mach number:

$$\frac{dV}{V} = \frac{dM^2}{2M^2\left(1 + \frac{\upsilon - 1}{2}M^2\right)}$$

Now, substituting this value in equation (18) and after integration:

$$\upsilon(M) \mp \theta = const$$

This can be written as,

$$\upsilon(M) + \theta = const = K_- \qquad \text{(Along the down running characteristic)}$$

$$\upsilon(M) - \theta = const = K_+ \qquad \text{(Along the up running characteristic)}$$

Note that our compatibility relations are now reduced to algebraic equations which hold only along the characteristic lines. In a general, steady inviscid supersonic flow compatibility equations are ordinary differential equations.

These results are used to determine the supersonic flow inside the nozzle and the determination of proper wall contour, so that shock wave does not appears inside the nozzle. The detailed design procedure is given in the following chapter.

# CHAPTER 4

# METHODOLOGY

Method of characteristics is a simple and efficient method for supersonic nozzle design. This method can be applied to one-dimensional unsteady flow, two and three-dimensional steady flows and axi-symmetric steady flow. The method of characteristics indicates the properties of a perfect gas in which continuous waves of small but finite amplitude are present as the gas flows supersonically, irrotationally and shock free in any passage.

The compatibility relation is given by,

$$\upsilon(M)+\theta \ = const = K_- \quad \textbf{(Along the down running characteristic)} \qquad \ldots (4.1)$$

$$\upsilon(M)-\theta \ = const = K_+ \quad \textbf{(Along the up running characteristic)} \qquad \ldots . (4.2)$$

Consider the following flow field, the flow properties at different locations are calculated using the following procedure.

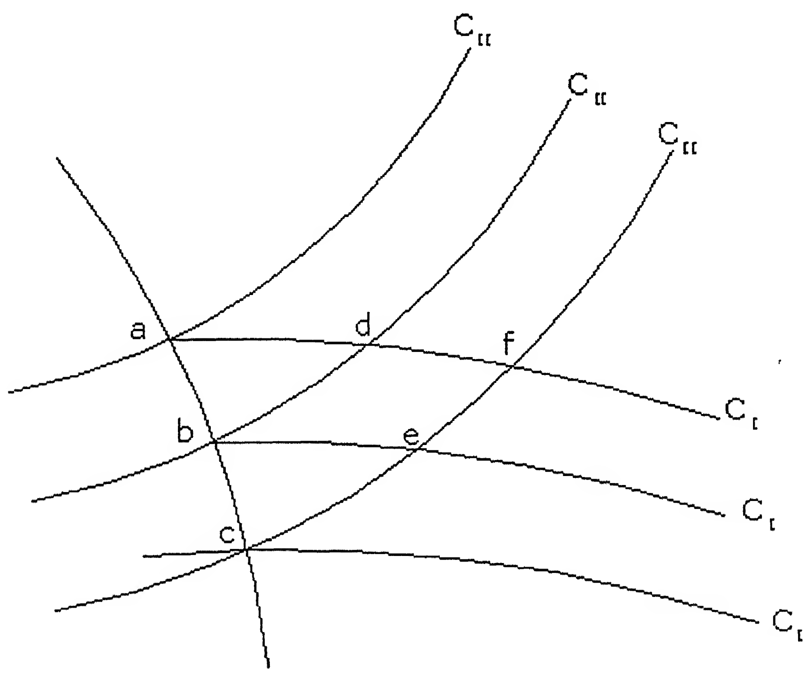


**Fig. 4.1 Grid points in the flow field**

With the given geometric coordinates and flow properties along a starting line, a number of initial points, such as points a, b, c, etc., are selected. From the values of θ and γ at each of these points, numerical value of characteristics $C_I$ and $C_{II}$ are determined. At point d, characteristic $C_I$ passing through point "a" intersects with characteristic $C_{II}$ passing through "b". The value of θ and γ at point "d" is calculated using the above compatibility equations.

For example, the two characteristic curves, which locate point "d", are

$$C_I = \theta_a + \upsilon_a = \theta_d + \upsilon_d$$

*and*

$$C_{II} = \upsilon_b - \theta_b = \upsilon_d - \theta_d$$

The resulting flow direction θ and the Prandtl-Mayer function $\upsilon$ at point "d" are determined by these characteristic values, $C_I$ and $C_{II}$

$$\theta_d = \frac{(\upsilon_a - \upsilon_b) + (\theta_a + \theta_b)}{2} = \frac{C_I - C_{II}}{2}$$

*and*

$$\upsilon_d = \frac{(\upsilon_a + \upsilon_b) + (\theta_a - \theta_b)}{2} = \frac{C_I + C_{II}}{2}$$

The Mach number and Mach angle corresponding to $\upsilon_d$ can be determined. Since the flow is isentropic, temperature and pressure at point "d" can be calculated from isentropic relations. Similarly, characteristic $C_I$ of point "b" and characteristic $C_{II}$ of point "c" are used to locate point "e". Point "f" has the same $C_I$ characteristic as point "a" and "d", and the same $C_{II}$ characteristics as points "c" and "e". The procedure is continued to other points in the flow field, and the complete net of characteristic curves describing the flow

pattern can be determined. Knowing the inclination, the characteristic lines allow the construction of the characteristic net in physical plane. This characteristic net gives the physical meaning of the flow.

## 4.1 Formulae

### 4.1.1 Atmospheric condition calculation

Nozzle exit Mach number and exit area depend on back pressure (or ambient pressure) of the nozzle. This back pressure value is not constant for rocket and missiles Similarly, temperature and density also vary with altitude. The variation of static temperature with geometric altitude consists of series of straight-line segments either isothermal or gradient. The static pressure decreases exponentially with increasing geometric altitude.

The variation of these thermodynamic parameters for different altitudes from Ref [27] is given below:

**Altitude <11000 m**

$$
\begin{aligned}
Temp &= 15.04 - (0.00649 * H) \\
Pres &= 101.29 * \left(\frac{temp + 273.1}{288.08}\right)^{5.256} \qquad \text{.......................... (4.3)} \\
Dens &= \frac{Pres}{(0.2869\,(Temp + 273.1))}
\end{aligned}
$$

**11000 m< Altitude < 25000 m**

$$
\begin{aligned}
Temp &= -56.46 \\
Pres &= 22.65 * e^{(1.73 - 0.000157 * H)} \qquad \text{.......................... (4.4)} \\
Dens &= \frac{Pres}{(0.2869\,(Temp + 273.1))}
\end{aligned}
$$

**Altitude > 25000 m**

$$Temp = -131.21 + 0.00299 * H$$
$$\Pr es = 2.488 * \left(\frac{Temp + 273.1}{216.6}\right)^{-11.388} \qquad \text{.....................................} (4.5)$$
$$Dens = \frac{\Pr es}{\left(0.2869\left(Temp + 273.1\right)\right)}$$

## 4.1.2 Specific heat ratio calculation

Normally, Specific heat ratio $\gamma$ is assumed constant for many applications, such as design of wind tunnel nozzle. In ordinary supersonic wind tunnel, the maximum temperature is around 300K. A gas is considered calorifically perfect up to 800 K. If the temperature is above 800 K, the vibrational degrees of freedom present in the gas will increase. Therefore, if the temperature is greater than 800 K but less than 1750 K, the gas can be considered as thermally perfect, which implies that, the gas properties are function of temperature alone. When the temperature exceeds 1750 K, the gas properties are functions of both temperature and pressure. These kinds of flows are called chemically reacting flows, because at this temperature the gas begins to dissociate. $O_2$ molecules dissociate to oxygen atoms and $N_2$ molecules dissociate to Nitrogen atoms. Due to this phenomenon, the amount of heat required to increase the temperature of one Kg mass will reduce. Specific heat ratio value ($\gamma$) depends on the number of degrees of freedom present in the gas. When the temperature exceeds 8000 K, ionization of gas atoms occurs. The atoms liberate electrons and this leads to ionized state of the gas. This kind of flow is called plasma dynamics.

In Rocket, the temperature at nozzle inlet is very high. Once the gas expands in the nozzle, temperature is reduced to very less value along the length of the nozzle and the ratio of static to total temperature margin is reduced to very less value at the exit. Due to

this drastic variation in temperature, it is very difficult to use constant $\gamma$ throughout the length of the nozzle.

The mixture specific heat is given by

$$C_P = \sum_i \alpha_i C_{P_i} \qquad \text{.......................... (4.6)}$$

Specific heat ratio is given by

$$\gamma = \frac{C_P}{C_P - R/C_{p\infty}} \qquad \text{.......................... (4.7)}$$

The maximum combustion temperature occurs when hydrocarbon fuels are mixed with just enough air so that all of hydrogen atoms form water vapor ($H_2O$) and all the carbon atoms form carbon dioxide. This particular nature of fuel and air is represented by a general chemical equation for complete combustion called stoichiometric equation.

This is a given by

$$C_x H_y + \left(x + \frac{y}{4}\right)\left(O_2 + \frac{79}{21} N_2\right) \rightarrow x\, CO_2 + \frac{y}{2} H_2O + \frac{79}{21}\left(x + \frac{y}{4}\right) N_2 \qquad \text{....... (4.8)}$$

$$m_i = n_i M_i$$

$$m = \Sigma m_i$$

$$m_{CO_2} = 44\, x$$

$$m_{H_2}O = \frac{y}{2} 18$$

$$m_{N_2} = \frac{79}{21}\left(x + \frac{y}{4}\right) 28$$

$$\text{Total mass} = 44\,x + 18\left(\frac{y}{2}\right) + \frac{79}{21}\left(x + \frac{y}{4}\right)28 \qquad \text{.......................... (4.9)}$$

$$\alpha_i \quad = \frac{m_i}{m} \qquad \text{..................... (4.10)}$$

$$C_{P_{CO_2}} = -0.0849 + 0.69384\left(T/_{100}\right)^{0.5} - 0.09326\left(T/_{100}\right) + 5.49954\times10^{-4}\left(T/_{100}\right)^2$$

$$C_{P_{H_2O}} = 7.9472 - 10.19667\left(T/_{100}\right)^{0.25} + 4.59728\left(T/_{100}\right)^{0.5} - 0.20549\left(T/_{100}\right)$$

$$C_{P_{N_2}} = 1.395 - 18.3139\left(T/_{100}\right)^{-1.5} + 38.311\left(T/_{100}\right)^{-2} - 29.3\left(T/_{100}\right)^{-3}$$

This formula can at least be used in the range 200 – 2000 K.

The specific heat capacity of the mixture $C_p$

$$C_p \quad = C_{P_{CO_2}}\,\alpha_{CO_2} + C_{P_{H_2O}}\,\alpha_{H_2O} + C_{P_{N_2}}\,\alpha_{N_2} \qquad \text{................................. (4.11)}$$

The characteristic gas constant value is a function of species concentration of the mixture. This is also calculated in the similar way.

$$R_{CO_2} \quad = R/_{M_{CO_2}}$$

$$R_{H_2O} \quad = R/_{M_{H_2O}}$$

$$R_{N_2} \quad = R/_{M_{N_2}}$$

$$R \quad = R_{CO_2}\,\alpha_{CO_2} + R_{H_2O}\,\alpha_{H_2O} + R_{N_2}\,\alpha_{N_2} \qquad \text{......................... (4.12)}$$

The free stream specific heat capacity is also calculated because this is used as a non-dimensional parameter. Here it is assumed that the atmospheric air contains 79 % of nitrogen and 21 % of oxygen.

Calculation of $C_p$

$$C_{P\infty} = \alpha_{O_2} C_{p_{O_2}} + \alpha_{N_2} C_{P_{N_2}}$$

$$C_{P_{O_2}} = 1\ 16975 + 6.28187 \times 10^{-4} \left(T/_{100}\right)^{1\ 5} - 5.5803 \left(T/_{100}\right)^{-1\ 5} + 7.4025 \left(T/_{100}\right)^{-2}$$

$$m_{O_2} = 32 \times 21$$

$$m_{N_2} = 28 \times 79$$

$$m = m_{O_2} + m_{N_2}$$

$$\alpha_{o_2} = {m_{o_2}}/_{m}$$

$$\gamma = \frac{C_P}{C_P - R/_{C_{p\infty}}}$$

So, the specific heat ratio ($\gamma$) at every characteristic zone is calculated from the local temperature of the flow.

### 4.1.3 Calculation of exit Mach number and exit area

The exit area and exit Mach number are calculated using isentropic relation. The nozzle inlet area, inlet static pressure, inlet static temperature and nozzle inlet Mach number are the input parameters of the program. The specific heat ratio value at the nozzle inlet is calculated from the inlet temperature. From the law conservation of mass, mass flow rate is constant through out the length of the nozzle.

Mass flow rate = ρA V

This can be modified using isentropic relations,

$$\text{Mass flow rate } (\dot{m}) = \frac{\sqrt{\frac{\gamma}{RT_0}}\ M\ P_0\ A}{\left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}} \qquad \text{..........................} (4.13)$$

From the given pressure, temperature and Mach number we can calculate the total temperature and total pressure at the inlet using isentropic relations.

$$\frac{T_{0i}}{T_\infty} = 1 + \frac{\gamma - 1}{2} M_i^2 \quad \text{......................... (4.14)}$$

$$\frac{P_{0i}}{P_i} = \left(1 + \frac{\gamma - 1}{2} M_i^2\right)^{\frac{\gamma}{\gamma - 1}} \quad \text{......................... (4.15)}$$

For Isentropic Flow, $P_{0i} = P_{0e}$

For correctly expanded flow $P_e$ = ambient pressure. Ambient pressure is calculated from the formulae given for different altitudes. The given nozzle design is shock free correctly expanded flow nozzle.

Assuming a value for Gamma, for example $\gamma_e$ =1.124

$$\frac{P_{0e}}{P_e} = \left(1 + \frac{\gamma_e - 1}{2} M^2\right)^{\frac{\gamma_e}{\gamma_e - 1}}$$

$$\left(1 + \frac{\gamma_e - 1}{2} M^2\right)^{\frac{\gamma_e}{\gamma_e - 1}} = \frac{\left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}}{\mathrm{Pres}/p_i}$$

$$M_e = \sqrt{\left[\left(\frac{\left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}}{\mathrm{Pres}/p_i}\right)^{\frac{\gamma_e - 1}{\gamma_e}} - 1\right] \frac{2}{\gamma_e - 1}} \quad \text{......................... (4.16)}$$

The exit temperature ratio is calculated from exit Mach number. Total temperature remains constant throughout the length. Hence, the exit temperature is calculated using this formula

$$T_e = \frac{T_{0i}}{T_{0e}/T_e}$$

From this, $T_e$ the local specific heat ratio value of the mixture can be calculated

Using iterative procedure, the exact value of gamma $(\upsilon)$ can be calculated. Once the value of gamma is calculated, then velocity of sound value can be calculated.

$$\text{Velocity of sound}\,(a) = \sqrt{\gamma\ R\,T} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.17)$$

Once the velocity of sound at the exit is calculated, the exit velocity can be calculated from this formula

$$\text{Exit Velocity}\,(V_e) = M_e\, a_e$$

Mach angle is calculated from the given formula,

$$\text{Mach angle}\,(\mu) = Sin^{-1}\left(\frac{1}{M}\right)$$

Area ratio at any section in the nozzle is given by the formula,

$$\frac{A_x}{A_*} = \left[\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \left(1+\frac{\gamma_x-1}{2}M^2\right)^{\frac{\gamma_x+1}{2(\gamma_x-1)}}\right]\frac{1}{M} \qquad \ldots\ldots\ldots\ \ldots\ldots (4.18)$$

From known values of $M_i$ and Ai, the throat area is calculated. Once the throat area is calculated using the area ratio relation, the exit area required for calculated exit Mach number can be determined.

### 4.1.4 Calculation of total deflection angle

When supersonic flow is turned through a convex corner, the flow expands resulting in an increase in velocity and drop in pressure .This expansion takes an angle up to 130.5 degree. The change in properties cannot occur abruptly across an expansion fan but rather

gradually across a series of waves emanating at the corner. Therefore, at the corner, flow is not isentropic. If the surface is a continuous expansion surface then flow is isentropic.

The Prandtl-Mayer function is given by the formula,

$$\upsilon = \sqrt{\frac{\gamma + 1}{\gamma - 1}} \tan^{-1} \sqrt{\frac{\gamma + 1}{\gamma - 1}\left(M^{2} - 1\right)} - \tan^{-1} \sqrt{\left(M^{2} - 1\right)} \qquad \ldots\ldots\ldots\ldots (4.19)$$

Prandtl-Mayer function is a similarity parameter. The derivation of this function is given in ref [7]. This is based on the assumption that the flow-turning angle is small.

Knowing the value of inlet and exit Mach number, the total flow-turning angle can be calculated. The design is a minimum length exhaust nozzle. This is achieved by placing sharp corners that generates centered simple or Prandtl – Mayer expansion fans at the nozzle entry, and also achieved by absorbing or canceling each of the characteristics upon their arrival at the opposite nozzle boundary, by means of change in slope of equal magnitude and sign.

Total flow turning from Nozzle inlet to exit is given by,

$$\begin{aligned} \upsilon = \sqrt{\frac{\gamma + 1}{\gamma - 1}} & \left[ \tan^{-1} \sqrt{\frac{\gamma + 1}{\gamma - 1}\left(M_{e}^{2} - 1\right)} - \tan^{-1} \sqrt{\frac{\gamma + 1}{\gamma - 1}\left(M_{i}^{2} - 1\right)} \right] \\ & - \left[ \tan^{-1} \sqrt{\left(M_{e}^{2} - 1\right)} - \tan^{-1} \sqrt{\left(M_{i}^{2} - 1\right)} \right] \end{aligned} \qquad \ldots. (4.20)$$

The amount of flow turning caused by the initial expansion fan must be precisely equal to the amount of opposite flow turning along the reminder of the boundary. Therefore, each corner must be equal to half of the total flow-turning angle associated with the expansion process. Drop in static pressure is one of the parameters to decide the length of the nozzle. This parameter is a function of exit Mach number.

Static Pressure ratio is given by,

$$\frac{P_e}{P_t} = \left[\frac{1+\frac{\gamma-1}{2}M_t^2}{1+\frac{\gamma-1}{2}M_e^2}\right]^{\frac{\gamma}{\gamma-1}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (4.21)$$

The ratio of Height at any given location is given by,

$$\frac{H_x}{H_t} = \frac{M_t}{M_x}\left[\frac{1+\frac{\gamma-1}{2}M_x^2}{1+\frac{\gamma-1}{2}M_t^2}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (4.22)$$

This is derived from simple isentropic relation.

Stream thrust function is one of the parameter to measure the total expansion of the flow. The stream thrust function is the parameter, which leads to the determination of mass flow rate specific thrust. This is important parameter for performance evaluation.

The stream thrust function is given by,

$$\frac{Sa_x - Sa_t}{Sa_e - Sa_t} = \frac{\dfrac{1+\gamma M_x^2}{M_x\sqrt{1+\frac{\gamma-1}{2}M_x^2}} - \dfrac{1+\gamma M_t^2}{M_t\sqrt{1+\frac{\gamma-1}{2}M_t^2}}}{\dfrac{1+\gamma M_e^2}{M_e\sqrt{1+\frac{\gamma-1}{2}M_e^2}} - \dfrac{1+\gamma M_t^2}{M_t\sqrt{1+\frac{\gamma-1}{2}M_t^2}}} \qquad \ldots\ldots\ldots\ (4.23)$$

### 4.1.5 Calculation of x and y co-ordinates

$$x = \frac{\left[\left(\frac{A}{A_*}\right)_i - \left(\frac{A}{A_*}\right)_{i-1}\right]A_*}{\tan(turn_{i,j})} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ (4.24)$$

$$y = \left(\frac{A}{A_*}\right)_i A_* \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ (4.25)$$

Uniform flow region length is given by,

$$x_u = \frac{A_e}{\tan\left(\sin^{-1}\left(\frac{1}{M_e}\right)\right)} \qquad \text{.....................} (4.26)$$

In contour nozzle, the flow at the exit is uniform. Therefore, after the last characteristic the flow must be uniform. The uniform flow length depends on last characteristic Mach number. Therefore ninety percent of the length of the nozzle is last characteristic length in hypersonic speeds. Hence, we can truncate the bottom portion of the nozzle after the last characteristic wave. The required flap length is given by,

$$\text{Flap length} = x - x_u$$

Still this length is too large for hypersonic vehicle. The length of the nozzle is finally optimized according to the values of drop in static pressure and gain in stream thrust function.

### 4.1.6 Intermediate point calculation

The intermediate points (characteristic intersection points) of characteristics are calculated using simple analytical geometry. From this the idea of characteristic and the physical appearance of the characteristics are known

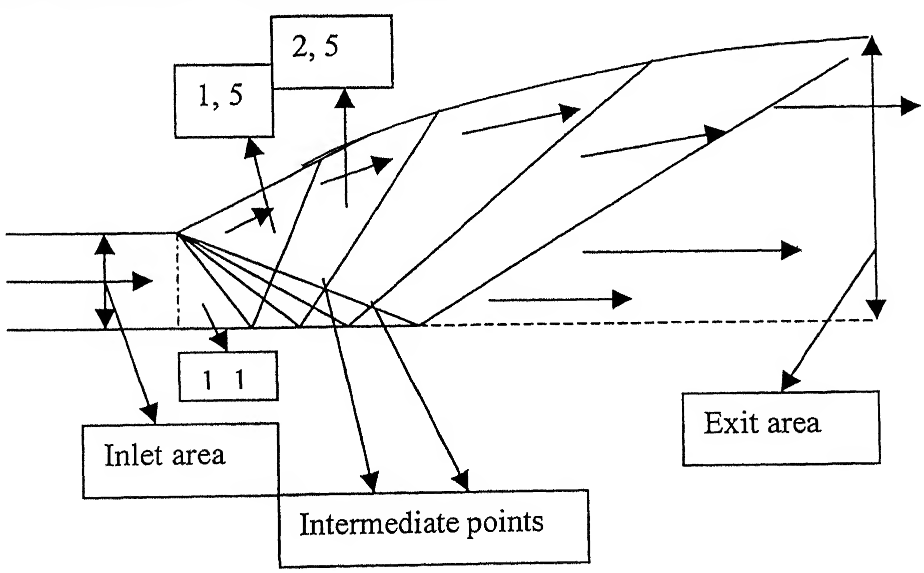


**Fig. 4.2 Single Expansion Ramp Nozzle with flap**

For second down running characteristic, the first intermediate point (1, 2) is calculated from the following method.

From two-point formula, second down running characteristic equation is written by,

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1}$$

$x_1 = 0$ and $y_1 = h\_i$

$$-xh\_i = x_2\left(y-h\_i\right)$$

$$xh\_i+x_2\,y-x_2h\_i = 0 \qquad \text{.....................} \; (4.27)$$

From one point and slope formula, first up running characteristic equation is written by,

$$y-y_1 = m\left(x-x_1\right)$$

$$y-y_1 = mx-m\,x_1$$

$$mx-y+y_1 - m\,x_1 = 0 \qquad \text{.....................} \; (4.28)$$

These two simultaneous equations are solved using Cramer's rule. The intermediate points at remaining locations are also calculated from this formula. Here, the assumption is that the characteristic is straight line in between two points.

### 4.1.7 Thrust calculation

The main objective of the nozzle design is to increase the thrust from the available pressure at the nozzle inlet of the vehicle. Thrust of the vehicle has two terms. (1) Momentum thrust is the main thrust in hypersonic vehicle. The main aim here is to increase the momentum as large as possible because, at the exit lot of available energy is wasted in the form pressure and temperature. This can be achieved by proper design of the nozzle contour. (2) Pressure thrust is an additional thrust in the nozzle in the case of under expanded nozzle.

Pressure thrust value is zero for perfectly designed nozzle. This means, all the available pressure energy is converted in to kinetic energy but this is not possible in hypersonic vehicle because the length required for correctly expanded flow is too large The length of the nozzle depends on various parameters. Therefore, the pressure thrust is also an important parameter in hypersonic vehicle unlike subsonic vehicle.

In general, thrust of the engine is given by

$$\mathbf{T} = \underbrace{(m_a + m_f)V_e - m_a V_i}_{\textbf{Momentum thrust}} + \underbrace{A_e(P_e - P_a) - A_i(P_i - P_a)}_{\textbf{Pressure Thrust}} \qquad \text{.............. (4.29)}$$

Static thrust of the engine is given by,

$$\mathbf{T} = (m_a + m_f)V_e + A_e(P_e - P_a)$$

Assume fuel air ratio is small,

$$\mathbf{T} = m_a\, V_e + A_e(P_e - P_a) \qquad \text{.............. (4.30)}$$

## 4.1.8 Force coefficient calculation

The extended surface in the nozzle exit produces axial and normal force at the exit of the nozzle. This is because of the flow angularity in the flow in the nozzle. These forces produce axial and normal moments in the nozzle exit. In Rocket and missile, these moments are very important to control the vehicle where aerodynamic controls are ineffective. In satellite, small rockets are used to put the satellites in correct orbit where the same principle is used. So, the value of these coefficients must be calculated precisely because this will lead to unstable vehicle.

Axial Force Coefficient is given by,

$$C_{F_x} = \int_0^{x} \left(P - P_\infty\right)_W \tan\theta_w \, d\,x \qquad \text{.............................. (4.31)}$$

Normal Force Coefficient along the wall is given by the formula,

$$C_{F_y,Wall} = \int_0^{x_e} \left(P - P_\infty\right)_W \, d\,x \qquad \text{.............................. (4.32)}$$

Normal Force Coefficient along the cowl (Flap) is given by the formula,

$$C_{F_y,cowl} = \int_0^{x_e} \left(P - P_\infty\right)_C \, d\,x \qquad \text{.............................. (4.33)}$$

Axial Moment Coefficient is given by,

$$C_{M_x} = \int_0^{x} \left(P - P_\infty\right)_W \tan\theta_w \left(y - 1\right) d\,x \qquad \text{.............................. (4.34)}$$

Normal Moment Coefficient along the wall is given by the formula,

$$C_{M_y,Wall} = \int_0^{x_e} \left(P - P_\infty\right)_W \, x \, d\,x \qquad \text{.............................. (4.35)}$$

Normal Moment Coefficient along the cowl is given by the formula,

$$C_{M_y,Wall} = \int_0^{x_e} \left(P - P_\infty\right)_c \, x \, d\,x \qquad \text{.............................. (4.36)}$$

Net Normal Moment Coefficient is then calculated from this formula,

$$C_{M_{x,net}} = C_{M_{x,Wall}} - C_{M_{x,cowl}} \qquad \text{.................................. (4.37)}$$

Here, pressure is normalized using dynamic pressure at exit.

Length (x) is normalized using throat area

Finally, force and moment coefficients are calculated for a given configuration. From analysis, it was found that this value is large at the initial portion of the nozzle after that this value is small because the static pressure of the nozzle is decreased to local ambient condition pressure. That is discussed in the next chapter.

# CHAPTER 5

# RESULTS AND DISCUSSION

## 5.1 Validation of results

The results obtained from Method of characteristics (MOC) numerical solution is verified with available hypersonic data given in Ref [4]. The results presented in this section are for two-dimensional hypersonic nozzles. This code is also used for axi-symmetric flow. This code incorporates the option to use different fuels in combustion chamber. Here it is assumed that the combustion-taking place in the combustor is complete combustion.

In Ref [4], the nozzle contour is designed using MOC and the flow is analyzed using Euler's code. In addition, the flow is considered as chemically reacting with vibrational non-equilibrium.

Input parameters for compared results are,

Nozzle inlet pressure = 154100 N/m$^2$.

Nozzle inlet temperature = 3001 K.

Nozzle inlet velocity = 5759 m/s.

Nozzle inlet area = 0.0508 m.

Design exit pressure equal to the ambient pressure at an altitude of 43.9 Km.

Fig. 5.1 shows the nozzle contour obtained from MOC and Ref [4] respectively. Here, the contour obtained from MOC technique is slightly under estimated compared to the Ref [4] result for frozen flow. However, the trend is same for both nozzle contours. This slight variation in results is because of species consideration. In the present work, the nozzle inlet has 79 percent nitrogen and 21 percent oxygen. Gases other than these two are neglected due to their negligible amount. In Ref [4], the results are obtained by solving all governing equations of flow by iterative procedure and the gas has nitrogen, oxygen, water, hydrogen and hydroxide.

In hypersonic vehicle, wall velocity distribution is an important parameter. The velocity distribution is a function of static pressure at the inlet of the nozzle. It should be calculated more accurately to predict the value of both the force and moment Co-efficient Fig 5.2 shows wall velocity distribution of both MOC and Ref [4] results. Here, both MOC and Euler's code results are almost same. Ninety percent of the velocity increment is gained in the initial portion of the nozzle. This is because of the sharp expansion corner placed at the inlet of the nozzle.

The effect of design exit pressure or altitude on nozzle contour is given in Fig. 5.3. Here for non-dimensional pressure value of 0.5, the exit area is increased to very high value, because that corresponds to very high altitude, where the exit Mach number is very high. In lower altitude, both the MOC and Ref [4] values are same. In this design, atmospheric conditions (pressure, temperature, density) are calculated using the standard formula given by NASA [27]. This may not be useful for very high altitude, where all the thermodynamic variables have lesser values. Therefore, this deviation in trend in the initial portion of the graph is mainly due to incorrect calculation of atmospheric pressure at very high altitude. The pressure at an altitude of 55 Km is equal to 48.022 $N/m^2$.

Fig. 5.4 shows the drop in static pressure along the length of the nozzle. Here, the static pressure is constant for a while after the corner expansion fan, since there is no characteristic wave in that zone where the flow is uniform. Therefore, the nozzle contour is a straight line up to the point where the first reflected characteristic wave meets the contour. However, in Ref [4], the pressure variation is smooth along the length of the nozzle. These results obtained from MOC are also verified with Euler's code.

## 5.2 Thermodynamic variables

The thermodynamic properties such as pressure, temperature and density are calculated using the formulae given by National Aeronautics and Space Administration (NASA) for different altitudes. Fig 5.5, Fig 5.6 and Fig 5.7 shows variation of temperature, pressure and density with altitude. In Fig 5.5, the temperature plot consists of series of straight-

line segments and constant temperature segments. In troposphere, the temperature reduces as altitude increases because the radiation produced by the earth surface reduces as altitude increases. Hypersonic vehicles prefer altitude from upper stratosphere to very high altitude for flight.

In Fig. 5.6, the pressure reduces exponentially as altitude increases. When altitude increases, mass of the substance in the atmosphere is reduced because of gravitational pull. For current generation flight, the materials available to withstand the load is in the range of 23,000 N/m$^2$ – 95,000 N/m$^2$. In troposphere, the density of the air is very high compared to stratosphere. If Mach number exceeds five, the loads acting on the vehicle is very high compared to available maximum allowable load. Therefore, hypersonic flight is not possible in troposphere.

In Fig. 5.7, density reduces exponentially with increase in altitude. Hypersonic flight is possible in stratosphere and above, because the density at this altitude is very less. However, hypersonic flight needs higher mass flow at inlet to sustain hypersonic flight. This is achieved by choosing larger stream tube area at the inlet with a series of oblique shock where the pressure loss is very less. Similarly, to expand the flow to hypersonic speed higher area at the nozzle exit is required. All these requirements are satisfied in hypersonic vehicle, when choosing the entire bottom surface of the vehicle as a thrust producing device.

## 5.3 Parameters that affect the nozzle performance

Fig. 5.8 shows the schematic diagram of single expansion ramp nozzle. Here, the uniform flow region is a zone where the flow properties are constant and has no waves Simple region has characteristic waves, which do not cross each other. If characteristic waves cross each other, the region is called non-simple region. Fig. 5.9 to 5.13 shows the effect of different parameters that affect the nozzle performance for inlet Mach number 1.5, 2.0, 2.5 at a constant exit Mach number 5. Here the specific heat ratio value is 1.24.

### 5.3.1 Area ratio

Fig. 5 9 shows the effects of area ratios on nozzle performance. The area ratio is a function of exit Mach number. Here, the points 1, 2, 3 denote the area ratio required for exit Mach number 5 for inlet Mach number 2.5, 2.0, 1.5 respectively. For inlet Mach number 1.5, the point after the initial expansion occurring near the wall (uniform region) has a Mach number of 2.84, with a corresponding area ratio of 3.5. This is a reasonable exit area with lesser length. The area ratio for exit Mach number 5 is 63.4, since this value is too large, hypersonic flight is not possible. In order to sustain hypersonic flight, the nozzle is truncated to get reasonable length and area.

When the inlet Mach number increases to high values, the uniform flow region length behind the sharp expansion corner also increases. This leads to a straight-line shape in the initial portion of the nozzle with large flow turning angle. Therefore, for a very high inlet Mach number the nozzle becomes straight-line with large angle in the initial portion of the nozzle.

### 5.3.2 Uniform flow region length

Fig. 5.10 shows the effect of uniform flow region lengths on overall nozzle length of the vehicle. This uniform flow region length is a function of exit Mach number of the nozzle. If the exit Mach number increases to hypersonic Mach number range, then the uniform flow region length is around 80 to 90 percent of the overall length of the nozzle, irrespective of the nozzle inlet Mach number. Slight increase in length causes both increase in weight and drag in hypersonic vehicle. Similarly if the nozzle inlet Mach number is very large (say M=5) then flap length also increases to a considerably large value. This produces additional drag. Therefore, in hypersonic flight both inlet and exit Mach numbers are important parameters.

### 5.3.3. Total flow-turning angle

Fig. 5 11 shows the effect of total flow turning angles on nozzle performance. Here, the point 3 denotes the total flow turning required for exit Mach number 5 where the inlet Mach number is 1.5. From the graph, the total deflection angle required for Mach 5 is 87 degree The flow turning at the initial simple expansion fan corner is one-half of the total flow-turning angle. Therefore, the initial expansion angle is 43.5 degree. This leads to a very large value of height, which produces a large drag. If the nozzle inlet Mach number is of higher value, (say M=2.5) then the initial expansion is around 26 degree. Therefore, this produces less drag compare to the previous case. In the case of higher nozzle exit Mach number requirements, the Mach number is increased to high value before it comes to nozzle inlet.

### 5.3.4. Nozzle static pressure drop

Fig. 5.12 shows the drop in static pressures along the length of the nozzle. Here, the point 1 shows the drop in static pressure value at uniform flow region. 90% of the pressure energy is converted into kinetic energy at this zone. Therefore, the remaining 10% of the available pressure takes the entire remaining length of the nozzle for expansion. This drop in pressure is also a function of back pressure (ambient pressure). If the back pressure is high enough then the drop in pressure in initial, expansion fan is reduced. Therefore, the amount of pressure energy converted to kinetic energy is also a function of exit Mach number.

### 5.3.5. Stream thrust function

Stream thrust function at a location is a measure of how much pressure energy is converted in to kinetic energy in the flow. So this function is used to measure the performance of the nozzle. Fig. 5.13 shows the stream thrust function with area ratio along the nozzle length. Here, point 1 denotes the stream thrust function at uniform zone. From this graph, more than 50% of the pressure energy is converted into kinetic energy in

the initial portion of the nozzle inlet because of sharp expansion corner. 60 percent of the stream thrust function value gives reasonable length. Therefore, for any supersonic nozzle design all the above parameters are to be considered to optimize the nozzle.

## 5.4 Effect of Gamma on Nozzle exit parameters

Specific heat ratio is a sensitive parameter in hypersonic speed range. This value depends on mass concentration of each species, temperature, and pressure. The nozzle flow in hypersonic flight has very high temperature. At this temperature, either gas or gas mixture will begin to dissociate. Oxygen would dissociate at the temperature above 2200 K. Nitrogen would dissociate at the temperature above 4500 K. Hence, up to 2500 K the gas is assumed as thermally perfect gas and the flow is frozen (the chemical composition of the exhaust is constant) with no dissociation. In this nozzle design, the chemical composition at the nozzle inlet is frozen.

Specific heat capacity ($C_p$) is one of the sensitive parameters in hypersonic nozzle design In ordinary wind tunnel supersonic nozzle design, the specific heat ratio is assumed constant because in wind tunnels nozzle the maximum temperature is below 800 K. For temperatures up to 800 K, the gas is assumed calorifically perfect. Once the temperature increases above 800 K, the vibrational degree of freedom of the diatomic gas increases. This will reduce the value of specific heat ratio.

Table 1 show the effect of specific heat ratio on nozzle parameters, where the input parameters are constant for all cases. The gamma value of 1.24 shows that the Mach number at exit is reasonably lesser compared to other two cases of gamma 1.33 and 1.4 respectively. If gamma increases, all the exit parameters Mach number, exit area and length are also increases. The assumption of higher value of gamma leads to incorrect value of exit Mach number that leads to over estimated nozzle contour. If the assumed gamma value is less, then this leads to under estimation of the nozzle contour. Therefore,

the only way to predict the correct nozzle contour is variable gamma design, where gamma is function of static temperature throughout the nozzle length.

The static temperature is not a constant value along the nozzle length. This reduces along the nozzle length when the flow begins to expand. The specific heat ratio is a function of temperature when the temperature exceeds 800 K. Therefore, the gamma value increases from a lesser value to a higher value along length of the nozzle because of reduction in vibrational degree of freedom, which makes the constant gamma assumption invalid for hypersonic nozzle design. The variable gamma is calculated along the length of the nozzle using the formula given in Ref [4]. Apart form this variable gamma of gas, the fuel used in the combustion chamber is also taken into account. Here, the combustion takes place in the combustor is complete. So, the stoichiometric equation is used to calculate the exact composition of the exhaust. The specific heat capacity value is calculated from the formula given in Ref [5]. The gamma value for propane fuel is also calculated from this formula and the corresponding results are given in the table.

Fig. 5.14 shows the effect of gamma value on nozzle contour for gas and propane fuel with both constant and variable specific heat ratio. Here the design exit pressure is equal to the ambient pressure at an altitude of 30000 m. The other input parameters are given below.

Nozzle inlet Mach No = 1.500

Nozzle inlet static temperature = 1800.000 K

Nozzle inlet static pressure = 306551.6401 $N/m^2$

The inlet height or area of nozzle = 0.600 m

Here, in the constant gamma case the gamma value of 1.33 gives good approximate value for exit area compared to other two cases. The nozzle is truncated to a lesser length where this approximate gamma gives best contour. In variable gamma gas, the air contains 79

percent nitrogen and 21 percent of oxygen. Since the gas composition is known, the specific heat ratio value for air at different temperature is calculated. Similarly, the propane fuel has three carbon atoms and eight hydrogen atoms. This fuel reacts with air and gives the products. This product composition is calculated using stoichiometric equation. Here, the exit area of propane fuel is slightly higher compare to variable gamma of gas Therefore, the correct value of exit area is predicted when the fuel is also taken into account.

## 5.5 Effect of back pressure

The exit area of the nozzle is a function of both nozzle inlet static pressure and ambient pressure. Fig. 5.15 shows the effect of back pressure or altitude on exit area or nozzle contour of the nozzle. The nozzle inlet pressure is constant for all three cases. The exit area for constant gamma value is large compared to variable gamma at very low altitude or high-pressure region. However, the difference in exit area for propane and variable gamma of gas are very less. For low-pressure region or very high altitude (>50 km) all three have considerable difference in exit area. Therefore, for hypersonic flight speed the specific heat ratio is an important factor.

If the back pressure increases then the Mach number at the nozzle exit reduces because of high backward force. This reduction in Mach number causes the reduction in exit area. For correctly expanded flow at that, altitude needs less exit area. The entire nozzle contour is modified when the back pressure increases. The higher exit area leads to over expansion and lesser exit area causes under expansion. In both the process, loss of thrust takes place, therefore the given nozzle contour is suitable only for a particular altitude. In higher back pressure region, the variation of exit area with pressure is less compared to lesser back pressure region (high altitude). The variation of nozzle contour is large in high altitude where the hypersonic flight is possible. The cruising altitude must be constant for hypersonic vehicles. Therefore selecting a cruising altitude for hypersonic flight is a difficult task for designers where all the parameters must be considered.

## 5.6 Variation of Nozzle length with number of characteristics

The supersonic flow is wave-dominated flow with infinite number of characteristics moving both right and left sides along the flow. However, it is not possible to consider infinite number of characteristics in computation. The number of characteristics is maximized depends on computer's efficiency and speed. Therefore, for simplicity finite number of characteristics is considered. The accuracy of the problem depends on number of characteristics.

Table 2 shows the variation of total length of the nozzle with number of characteristics for constant gamma for given inputs. If the number of characteristics is less then the nozzle length is also less. If the number of characteristics increases then the total length of the nozzle is increased. However, after certain number of characteristics the length is almost remains constant. Therefore, the less angle (<=1 degree) between the characteristics gives results with good accuracy. For lesser angles, impact on nozzle length is very less. Table 3 shows the variation of nozzle length with number of characteristics for variable gamma.

The reduction in number of characteristics leads to increase in the angle between two successive Mach lines. This large angle shows shock wave instead of Mach wave because the changes in properties across the Mach wave are very small. The reduction in number of characteristics leads to shock instead of Mach line in the flow. Therefore, the number of characteristics is assumed as large as possible in the total deflection angle. This leads to accurate solution.

## 5.7 Results for test case

The following results are for two-dimensional nozzles. In Method of characteristic technique both two dimensional and axi-symmetric nozzle are same because each characteristic passes and reflects on its own plane. Therefore, the only difference in axi-symmetric flow the input is radius, whereas in 2-d nozzles it is the inlet area. Here, the

output file from the program is given where inlet Mach number, inlet area, inlet pressure and temperature are inputs along with altitude. The corresponding results in graphical form are also given figures from Fig. 5.16 to Fig. 5.31.

The one general output file for variable gamma for test case is given below:

Atmospheric conditions for 30000.000 m altitude

Atmospheric temperature= 231.490 K

Atmospheric pressure = 1161.181 N/m$^2$

Atmospheric density = 0.017 kg/m$^3$

**INPUTS**

Nozzle inlet Mach no = 1.500

Nozzle inlet Mach angle = 0.7297 rad

Nozzle inlet static temperature = 1800.000 K

Nozzle inlet total temperature = 2410.243 K

Nozzle inlet static pressure = 306551.640 N/m$^2$

Nozzle inlet total pressure = 1081483.876 N/m$^2$

The inlet height or area of nozzle = 0.600 m

Separation angle in radians = 0.0017 rad

**OUTPUTS**

Nozzle inlet local velocity of sound = 821.753 m/s

Nozzle inlet velocity = 1232.629 m/s

Required Mach no at exit = 5.485

Exit Mach angle = 0.183 rad

Exit temperature = 346.089 K

Exit velocity of sound = 373.297 m/s

Exit velocity = 2047.732 m/s

Throat area = 0.504 $m^2$

Area ratio $a/a_*$ at inlet=1.189

Area ratio $a/a_*$ at exit =37.580

Exit area of the nozzle =31.598 $m^2$

Total flow turning angle = 1.201 rad

Total flow turning angle = 68.852°

Total number of characteristics = 345

First characteristics angle = 0.000462 rad

Flow turning angle at inlet = 0.000 rad

Mass flow rate = 452.670 Kg/s

Nozzle overall length = 118.771 m

Uniform flow region length = 102.246 m

Flap length required for desired exit Mach number = 16.524 m

Fig. 5.15 shows the nozzle contour for inlet Mach number 1.5 where the exit Mach number is 5.5. From this, the inference is that, the initial expansion sharp corner angle is very large because the exit Mach number is very large also; the overall length of the nozzle is equal to 118 m. Therefore, from the above data, the flap length required for very high Mach number is very small compared to the overall length. The initial exit area at the inlet for this nozzle is very large compare to large inlet Mach number conditions. This large area at the initial portion causes large amount of drag. During such situations, conical nozzle yields good results compare to contour nozzle. In order to get such a high Mach number at exit through contour nozzle design, the nozzle inlet Mach number also should be high. This can be achieved by increasing the nozzle inlet area. Finally, the length of the nozzle is optimized using stream thrust function.

Fig 5.16 and Fig. 5.17 show the actual and enlarged view of intermediate points. This gives an idea of how the flow properties vary across the characteristic wave The flow properties variation is large in the initial portion of the nozzle compared to aft region of the nozzle This means the change in properties or expansion takes rapidly in the initial portion of the nozzle. Here the characteristic wave is straight line between any two points. However, in reality this is not a straight line because the local temperature is varying at every point in the flow.

Fig. 5.19 and Fig. 5.20 show the drop in static pressure and static temperature along the length of the nozzle respectively. Here the drop in pressure and temperature is very large in the initial portion compared to aft region of the nozzle. Here the figures show that at the nozzle inlet the drop in pressure is very large because of sharp expansion corner used at the inlet of the nozzle. These changes in properties are given in Fig. 5.17 also. Therefore, we can truncate the nozzle at certain location so that the reasonable exit Mach number is obtained at the exit. This can be optimized using stream thrust function.

Fig. 5.21 and Fig. 5.22 show the variation of stream thrust function with area ratio and length of the nozzle respectively. Here most of the available pressure energy is converted into kinetic energy in the initial portion of the nozzle. In hypersonic flight drag is a main parameter, which depends on both exit area and length of the nozzle. Therefore, both length and height are optimized using this parameter. Five times the inlet area of exit area gives reasonable length and height. The length of the nozzle also depends on the exit Mach number requirement.

Fig. 5.23 and Fig. 5.24 show the axial and normal force coefficient respectively. Here the axial and the normal force coefficients are normalized using exit dynamic pressure. Both axial and normal force coefficients are very large at the initial portion of the nozzle and this variation is small as the length increases. This axial force leads to additional lift in the nozzle surface, which produces pitching moment.

Fig. 5.25 and fig. 5.26 show the axial and normal moment coefficient along the length of the nozzle respectively. The stability parameters depend on the value of both axial and normal moment coefficients. The incorrect prediction of these coefficients leads to unstable configuration. This predicted value of both axial and normal force coefficients is suitable for a particular altitude. Therefore, in order to fix certain static margin for stability, these values must be calculated precisely.

Fig. 5.27 to Fig. 5.29 shows the variation of different types of thrust along the nozzle length. From Fig. 5.28, at the nozzle inlet the pressure thrust is large because at the inlet available pressure energy is large. This pressure energy is converted into kinetic energy when the flow expands down stream of the flow. Fig. 5.29 shows the variation of net static thrust along the length. The net thrust value at the inlet is very less. However, after the sharp expansion this thrust is increased to a larger value. After that, the increase in net thrust is very less. Therefore, the length is truncated after the sharp expansion depending on required Mach number at the exit.

Fig. 5.30 and Fig. 5.31 shows the truncated nozzle contours for given inlet conditions. Here the nozzle is truncated where the stream thrust function is 70 percent at the nozzle wall. The overall nozzle length for truncated nozzle is 3.8147 m and the exit area is 2.0945 m. The flap length required for truncated nozzle is 2.7 m. This gives the reasonable length and exit area compared to the full length and area of the nozzle. The exit Mach number for truncated Nozzle is 3.3818.

In the case of full nozzle, the exit Mach number is 5.5 where the length of the nozzle is 118.771 m and area is 31.599 $m^2$. This higher value of both length and area is not possible in hypersonic vehicles. Therefore, the nozzle is truncated depending on the required exit Mach number of the vehicle. The stream thrust function value around 60 percent, gives the reasonable exit area and exit Mach number. However, the truncated nozzle has flow with some angularity.

## 5.8 The results obtained from Euler's code

The results obtained from the MOC are verified with Euler's code results of inviscid flow In Euler's code, the contour of the Scramjet nozzle is designed using the contour obtained from present MOC method, and then the flow through the nozzle is analyzed by solving all the governing equations for given input parameters. Here, the specific heat ratio gamma is constant throughout the length of the nozzle. The MOC and the Euler's code results at exit are shown in Table 4.

The input and the output from the Method of characteristic numerical method for constant gamma are given below.

### Free stream conditions

| | |
|---|---|
| Altitude | =30000 m |
| Free stream temperature | =231.49 K |
| Free stream pressure | =1161.180 N/m$^2$ |
| Free stream density | =0.0174 Kg/m$^3$ |

### Inlet Conditions

| | |
|---|---|
| Nozzle inlet Mach no | =1.5 |
| Inlet temperature | =1800 K |
| Inlet pressure | =306551.640 N/m$^2$ |
| Inlet density | =0.593 Kg/m$^3$ |
| Inlet velocity | =1243.352 m/s |
| The inlet area | = 0.600 m |

**Exit Conditions**

| | |
|---|---|
| Nozzle exit Mach no | = 5.205 |
| Exit temperature | = 451.256 K |
| Exit pressure | =1161.180 N/m$^2$ |
| Exit velocity | = 2160.1158 m/s |
| Exit area of the nozzle | =22.857 m$^2$ |

Fig. 5.32 and Fig. 5.33 show the Mach number and pressure distribution inside the nozzle for hypersonic vehicle. Fig. 5.32 gives a good Mach number distribution compared to Method of characteristic results. Here each color zone corresponds to one particular Mach number. These zones are similar to the characteristics net explained in Fig. 5.18 where the Mach wave is straight line in between two points. However, in reality it is not a straight line. For simplicity, the Mach waves are assumed straight line in between two intermediate points. This Mach number distribution is for 5000 iterations and it takes 1 full day for 5000 iterations. However, this can be solved with in few minutes using Method of characteristic technique.

The drop in static pressure is given in Fig. 5.33. Here the drop in pressure is large in the initial portion of the nozzle. The aft portion of the blue color region shows the uniform flow in the nozzle where the pressure is constant. Fig. 5.34 shows the comparison of Mach number distribution along the nozzle wall for both MOC and Euler's code results. Here the MOC result shows the gradual variation of Mach number along the wall compare to Euler's code computational results.

In the initial portion, both curves are same because at this portion the expansion takes place rapidly. Therefore, the difference in value at each grid is large. However, in the down stream the expansion is small compared to initial portion. Large number of grids in the aft portion leads to waste of time. Less number of grids in the aft portion of the nozzle gives uniform variation of Mach number and flow properties.

## 5.9 Important fact

The designed nozzle is single sided and single expansion ramp nozzle. Here a flap is used for reflecting the characteristics and it is used to vary the exit conditions for over and under expanded cases. In the case of double-sided symmetric nozzle one-half of the nozzle inlet area in taken in to consideration for contour design. Here, the flow is symmetric in both sides along the length of the nozzle. This is not used in hypersonic vehicle because this produces enormous drag compare to net thrust. Therefore, in order to design a symmetrical nozzle the inlet area of the program should be one-half of the total inlet area of the nozzle.

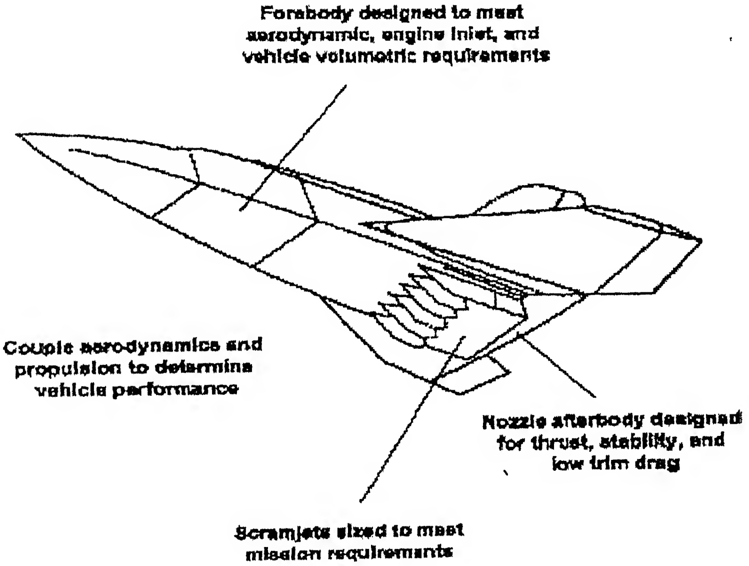


**Fig. 5.35 NASP (USA) hypersonic vehicle**

The current generation vehicles are mostly single sided conical nozzle. The conical nozzle exit has the flow with some angularity. Therefore, considerable loss in thrust takes place in such nozzles Fig. 5.35 shows the conventional National Aero Space Plane (NASP) hypersonic vehicle, which is a single sided expansion nozzle.

The contour nozzle has considerable axial flow at the exit of the nozzle because the nozzle is truncated at certain location. The flap is used for reflecting the characteristics or Mach waves. During off design conditions the flap can be moved up and down to adjust the flow in the nozzle exit to a correctly expanded flow. However, deflecting the flap at hypersonic Mach number is a very difficult task. The following Fig 3.35 shows the single expansion ramp nozzle.

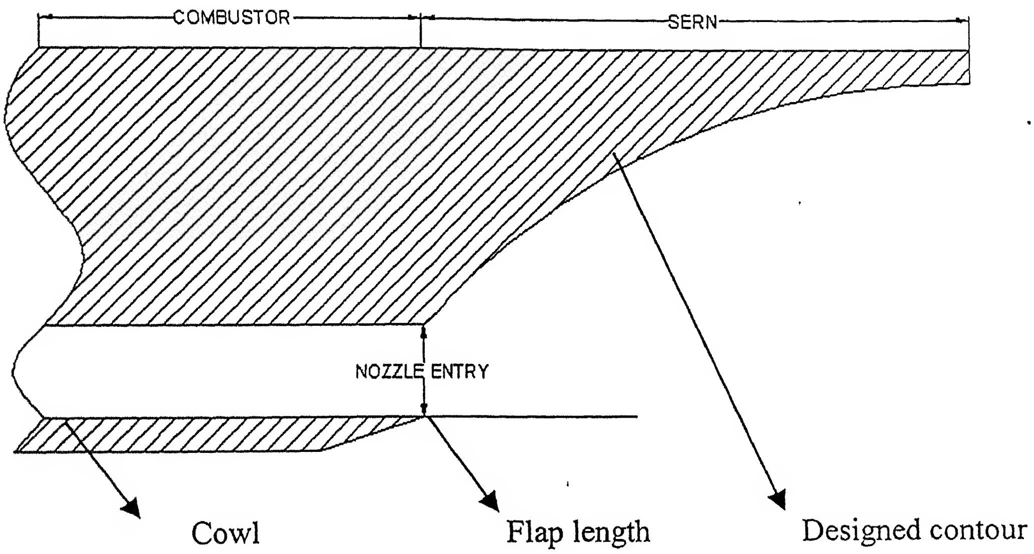


**Fig. 5.36 Single expansion ramp nozzle**

Here at the nozzle exit the flow is under expanded because the initial correctly expanded flow nozzle is truncated at some reasonable length. Therefore, it should have some expansion waves at the exit. However, change in property across this wave is small compared to initial expansion. It too has flow with certain angularity at the exit. However, this nozzle is efficient compared to conical nozzle because at the initial portion contour nozzle has more area for expansion whereas in conical nozzle, this area is less.

# CHAPTER 6

# CONCLUSION AND REMARKS

The method of characteristics computer program for designing and analyzing the Scramjet nozzles has been developed. This program has also incorporated chemical composition of air and fuel used in the combustion chamber for calculating the specific heat ratio, where complete combustion is assumed. The calculated chemical composition in the nozzle inlet is assumed constant throughout the length of the nozzle. The temperature and pressure along the wall at different locations are calculated, using which the force coefficients are calculated. Provision is provided for calculating the nozzle contour at different altitudes. This code can also be used for axi-symmetric nozzles. The code was used here to design a generic two-dimensional hypersonic nozzle based on arbitrarily selected flight conditions. This program is used for static temperature range up to 2500 K with best accuracy. However, it can be used for temperature up to 3000 K with a slight lesser accuracy. An evaluation of the various effects studied leads to the following conclusions.

1. Scramjet nozzles have good performance estimates when gamma has variable values rather than having various assumed constant values. Here flow is assumed to be frozen from the nozzle inlet to exit. The exhaust gas used in the combustion chamber also influences the performance of the nozzle.

2. Here, the chemical composition is assumed constant throughout the nozzle. If the temperature is very high, the chemical composition varies across each wave while expansion. Therefore, up to 2500 K, the gas is assumed as a function of temperature.

3. In the case of Scramjet nozzles, the separation angle for each characteristic should be less than one degree. In our case, 0.25° gives good results.

4 If the Mach number is of low supersonic at the inlet, while at the exit the Mach number is of hypersonic, then the initial portion of the nozzle has a larger height. In such situation, two single expansion ramp nozzles are preferred.

5. Most of the available pressure and temperature are converted into kinetic energy in the initial portion of the nozzle. Therefore, stream thrust function value of sixty percent gives reasonable height or area and length for hypersonic vehicles.

6. The angularity of flow in the nozzle wall region produces additional lift in the nozzle portion, which produces pitching moment about the centre of gravity of the vehicle. Therefore, these parameters are used to calculate vehicle static margin or trim requirements of the vehicle.

7. Momentum and pressure thrust along the axial location of the nozzle at different locations are calculated. The net thrust is calculated by adding up the momentum and pressure thrust.

# RESULTS

## TABLE 1

### Variation of Exit condition with Specific Heat Ratio

**INLET CONDITIONS**

| | |
|---|---|
| Altitude of flight | 30000 m |
| Atmospheric Temperature | 231.49 K |
| Atmospheric Pressure | 1161.18042 $N/m^2$ |
| Atmospheric Density | 0.017476 $kg/m^3$ |
| Inlet Mach number | 1.500 |
| Nozzle Inlet Static Temperature | 1800.00 K |
| Nozzle Inlet Static Pressure | 306551.640140 $N/m^2$ |
| Inlet Area of the Nozzle | 0.60 $m^2$ |

| Specific heat ratio (Gamma) | Nozzle Outlet Conditions |
|---|---|
| Gamma=1.4 | Exit Mach number 5.537470<br>Exit Temperature 365.919597 K<br>Exit Velocity 2123.289318 m/s<br>Exit Area of the Nozzle 19.346021 $m^2$<br>Nozzle Overall Length 125.348746 m<br>Required Flap Length 19.981207 m |

| | |
|---|---|
| Gamma=1.33 | Exit Mach number 5.204734<br>Exit Temperature 451.256437 K<br>Exit Velocity 2334.298584 m/s<br>Exit Area of the Nozzle 22.857215 $m^2$<br>Nozzle Overall Length 139.261340 m<br>Required Flap Length 22.511033 m |
| Gamma=1.24 | Exit Mach number 4.775643<br>Exit Temperature 611.751399 K<br>Exit Velocity 2228.287071 m/s<br>Exit Area of the Nozzle 29.004528 $m^2$<br>Nozzle Overall Length 162.186408 m<br>Required Flap Length 26.740636 m |
| Correct prediction<br>Variable gamma | Exit Mach number 5.485522<br>Exit Temperature 346.089108 K<br>Exit Velocity 2047.732415 m/s<br>Exit Area of the Nozzle 18.959363 $m^2$<br>Nozzle Overall Length 118.771246 m<br>Required Flap Length 16.524270 m |
| Correct prediction<br>Variable gamma<br>for propane | Exit Mach number 5.321639<br>Exit Temperature 379.071917 K<br>Exit Velocity 2072.658005 m/s<br>Exit Area of the Nozzle 20.431952 $m^2$<br>Nozzle Overall Length 123.218631 m<br>Required Flap Length 16.540835 m |

# TABLE 2

## Variation of nozzle length with number of characteristics for constant gamma

### INLET CONDITIONS

| | |
|---|---|
| Altitude of flight | 30000 m |
| Atmospheric Temperature | 231.49 K |
| Atmospheric Pressure | 1161.18042 $N/m^2$ |
| Atmospheric Density | 0.017476 $kg/m^3$ |
| Inlet Mach number | 1.500 |
| Specific Heat Ratio | 1.3300 kJ/kg |
| Nozzle Inlet Static Temperature | 1800.00 K |
| Nozzle Inlet Static Pressure | 306551.640140 $N/m^2$ |
| Inlet Area of the Nozzle | $0.60m^2$ |

### EXIT CONDITIONS

| | |
|---|---|
| The exit Mach number | 5.204734 |
| Exit Temperature | 451.256437 K |
| Exit Velocity | 2160.115896 m/s |
| Exit area of the nozzle | 22.857215 $m^2$ |

| Number of Characteristics | Exit condıtions |
|---|---|
| Total number of characteristics 20 | Nozzle overall Length 132.666028 m<br><br>Flap Length 15.915722 m |
| Total number of characteristics 39 | Nozzle overall Length 135.729271 m<br><br>Flap Length 18.978964 m |
| Total number of characteristics 76 | Nozzle overall Length 137.584508 m<br><br>Flap Length 20.834202 m |
| Total number of characteristics 151 | Nozzle overall Length 138.610820 m<br><br>Flap Length 21.860513 m |
| Total number of characteristics 375 | Nozzle overall Length 139.261340 m<br><br>Flap Length 22.511033 m |

# TABLE 3

## Variation of nozzle length with number of characteristics for variable gamma

### INLET CONDITIONS

| | |
|---|---|
| Altitude of Flight | 30000 m |
| Atmospheric Temperature | 231 49 K |
| Atmospheric Pressure | 1161.18042 $N/m^2$ |
| Atmospheric Density | 0.017476 $kg/m^3$ |
| Inlet Mach number | 1.500 |
| Specific heat ratio | 1.33 KJ/kg-K |
| Nozzle Inlet Static Temperature | 1800.00 K |
| Nozzle Inlet Static Pressure | 306551.640140 $N/m^2$ |
| Inlet Area of the Nozzle | 60.000 $cm^2$ |

### EXIT CONDITIONS

| | |
|---|---|
| The exit Mach number | 5.204734 |
| Exit Temperature | 451.256437 K |
| Exit Velocity | 2160.115896 m/s |
| Exit area of the nozzle | 22.857215 $m^2$ |

| Number of Characteristics | Exit conditions |
| --- | --- |
| Total number of characteristics 19 | Nozzle overall Length 113.082561 m<br>Flap Length 11.481438 m |
| Total number of characteristics 36 | Nozzle overall Length 115.830752 m<br>Flap Length 13.788870 m |
| Total number of characteristics 70 | Nozzle overall Length 117.390839 m<br>Flap Length 15.218479 m |
| Total number of characteristics 139 | Nozzle overall Length 118.238067 m<br>Flap Length 16.016498 m |
| Total number of characteristics 346 | Nozzle overall Length 118.771246 m<br>Flap Length 16.524270 m |

# TABLE 4

## COMPARISON OF RESULTS

| PARAMETERS | MOC RESULT | EULER'S CODE RESULT | PERCENTAGE OF ERROR |
|---|---|---|---|
| Mach number At exit | 5.205 | 5.189 | 1.067 |
| Static pressure at exit | 1161.168 N/m2 | 892.245 $N/m^2$ | .0877 |

# List of graphs

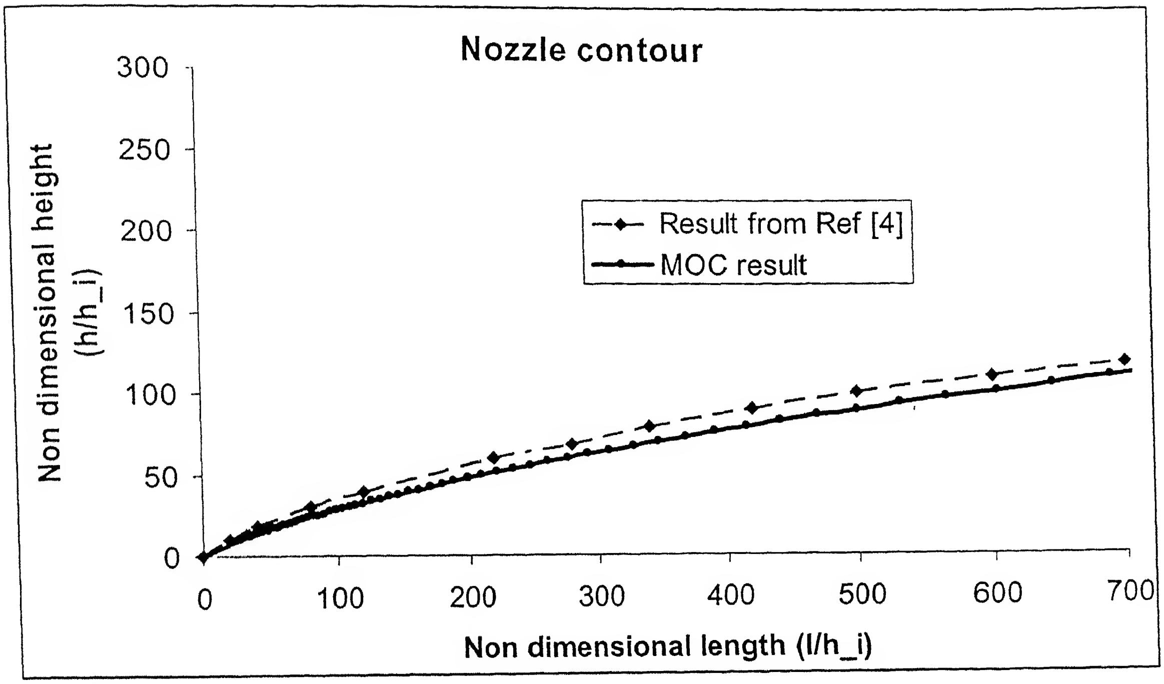


**Fig. 5.1 Nozzle contour for frozen flow**

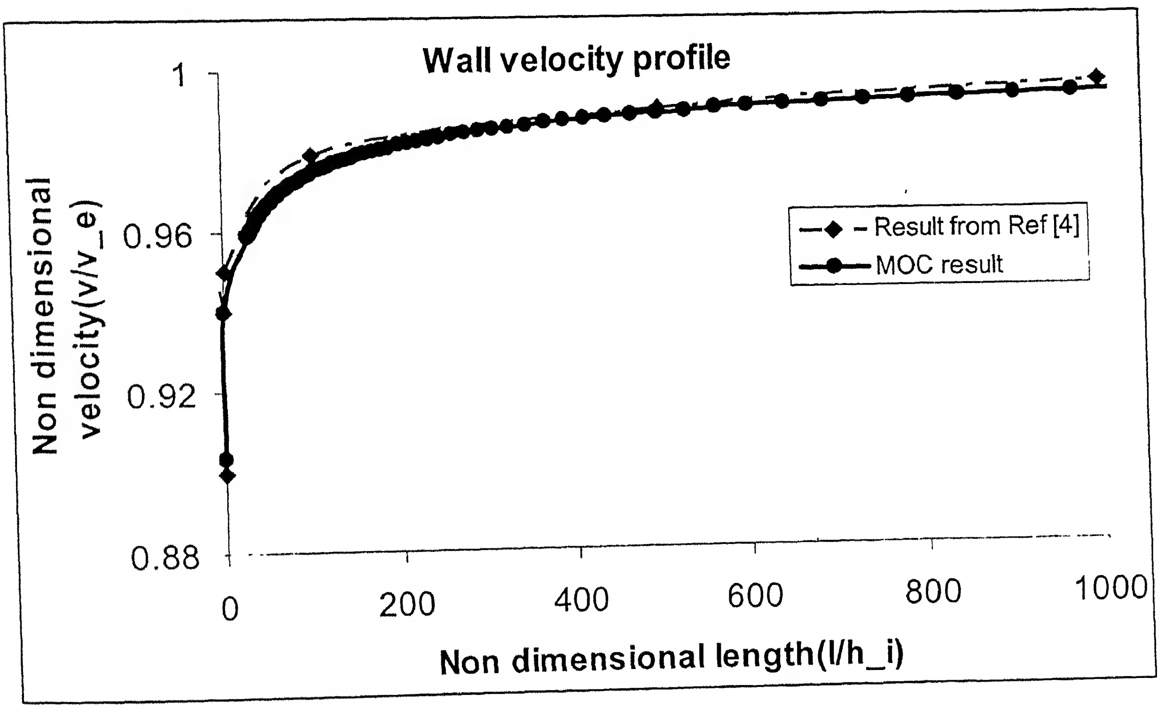


**Fig. 5.2 Wall Velocity profile**

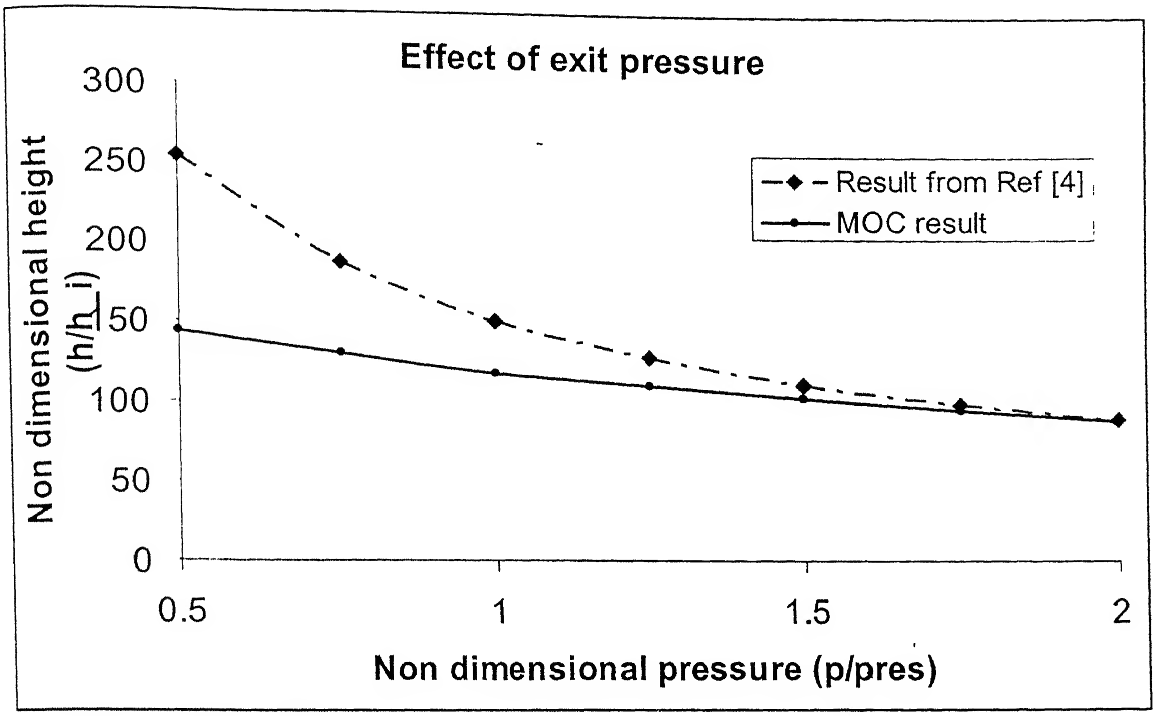


**Fig. 5.3 Effect of Exit Pressure**

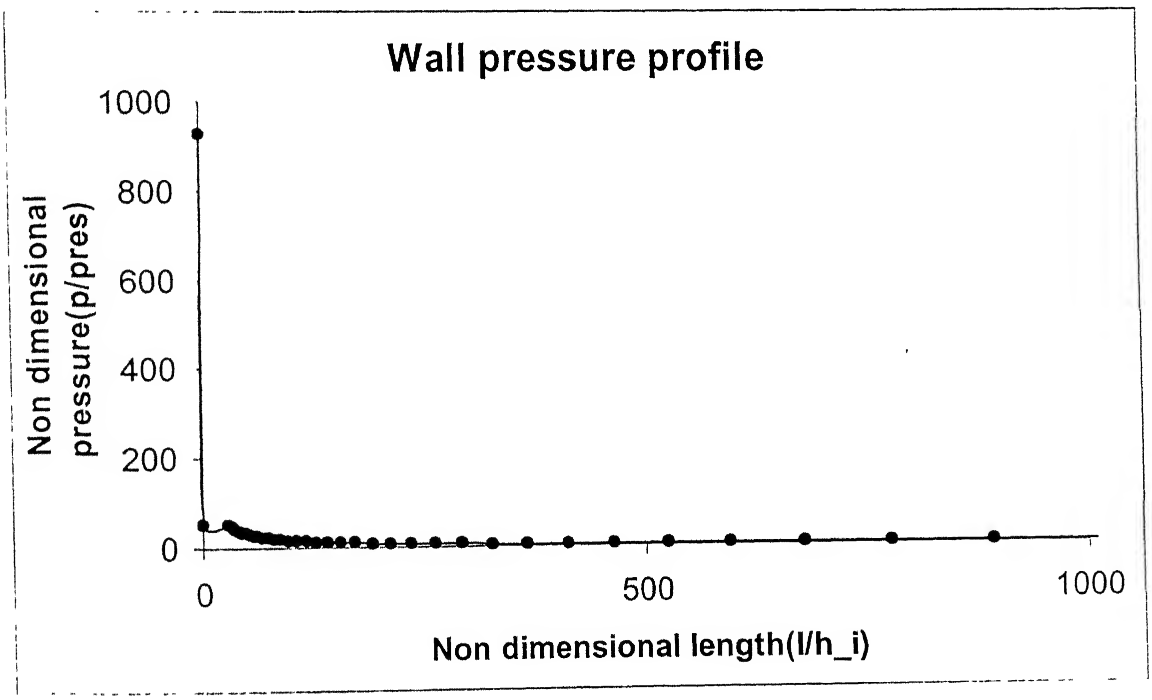


**Fig. 5.4 Drop in Static pressure along the wall**

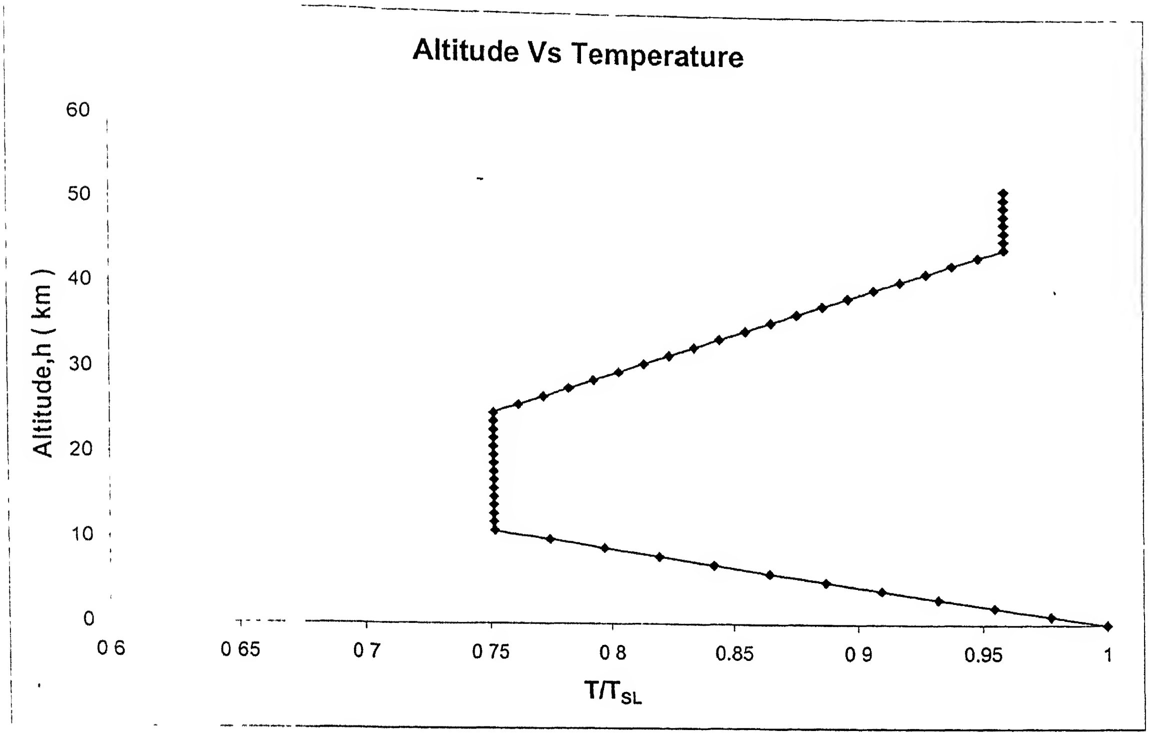


**Fig. 5.5 Variation of temperature with altitude**

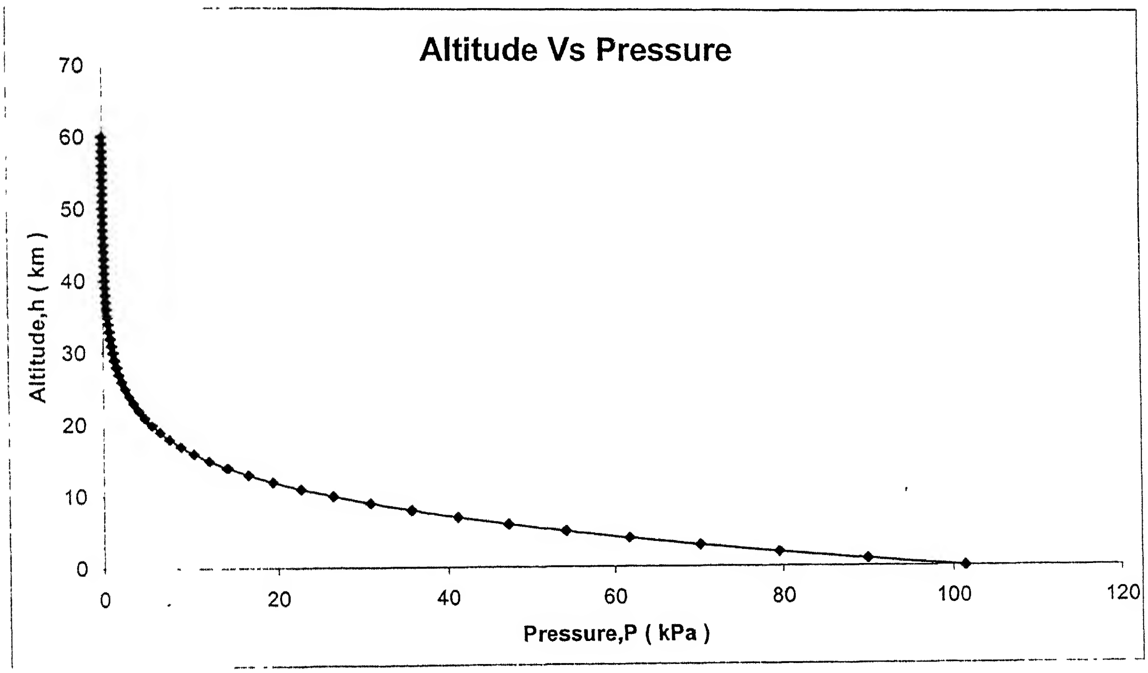


**Fig. 5.6 Variation pressure with altitude**

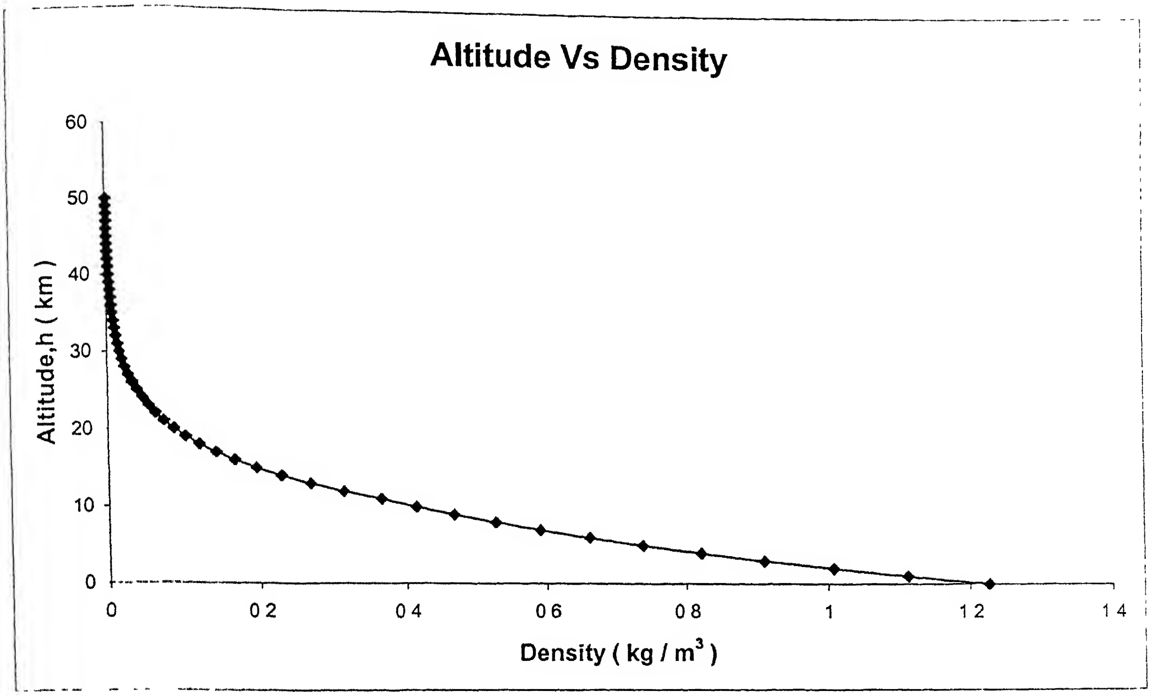


**Fig. 5.7 Density variation with Altitude**

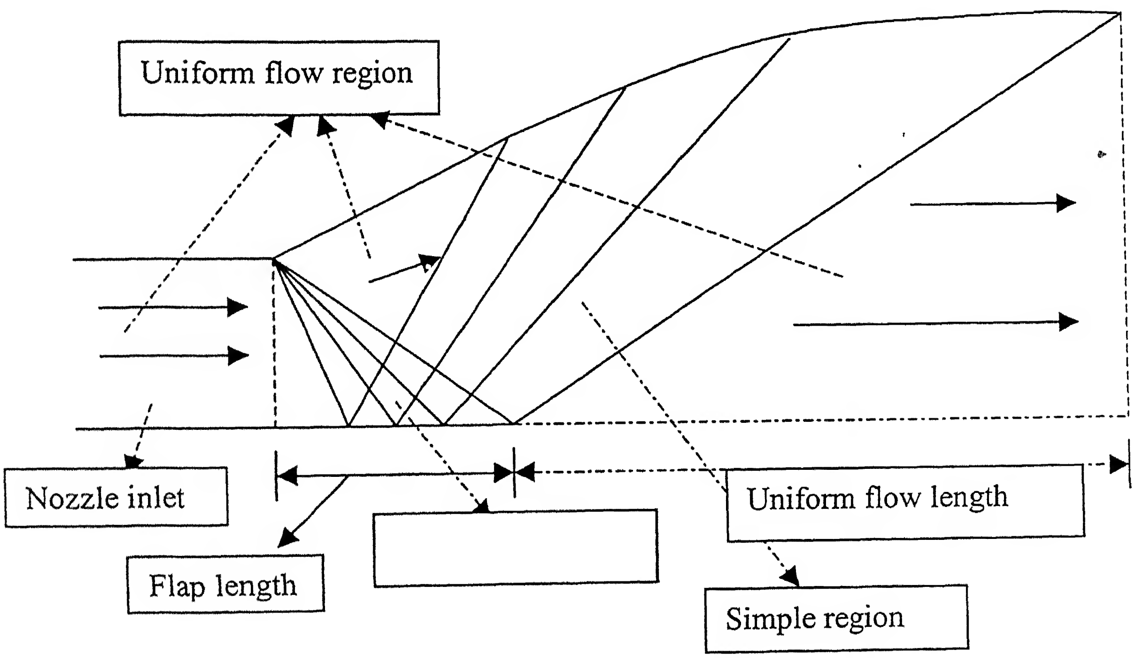


**Fig. 5.8 Characteristic diagram**

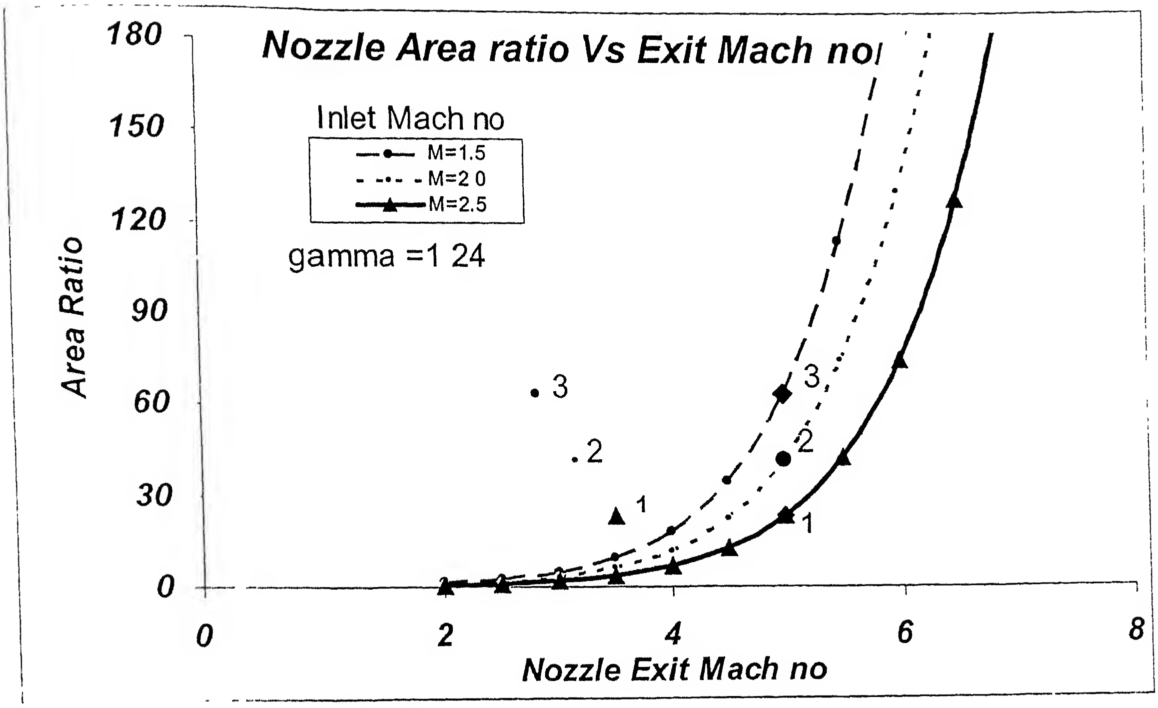


**Fig. 5.9 Effect area ratio on nozzle performance**

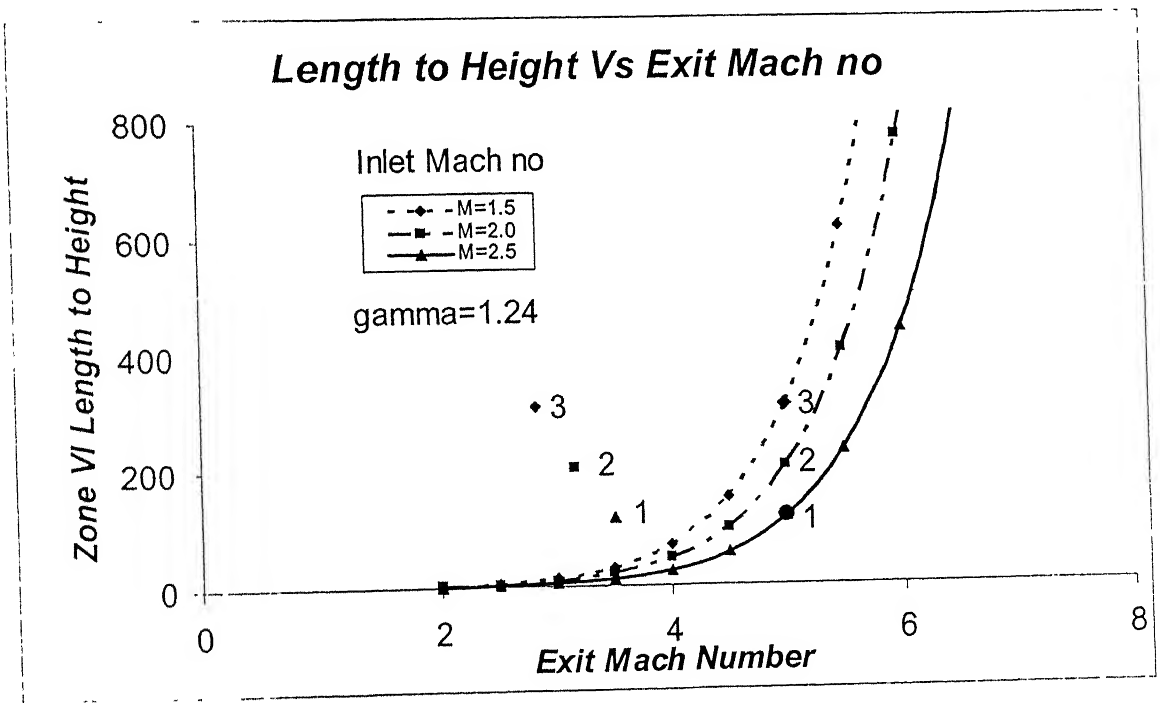


**Fig. 5.10 Effect of uniform flow region length on nozzle length**

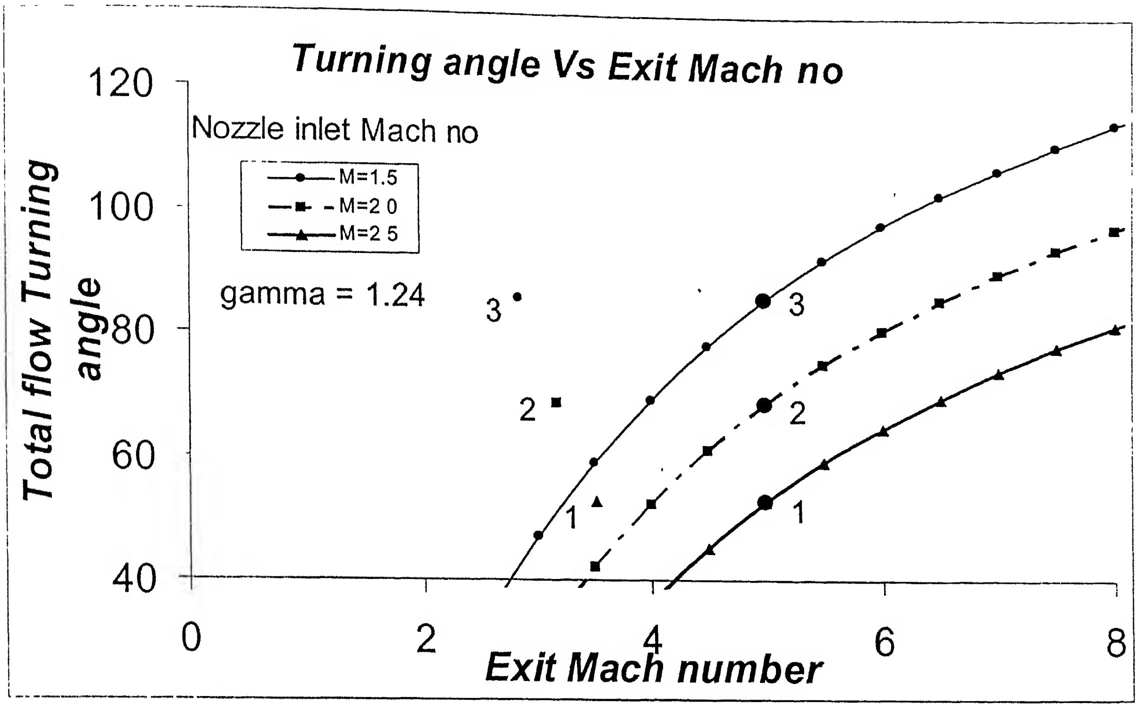


**Fig. 5.11 Effect of Total flow turning angle on nozzle performance**

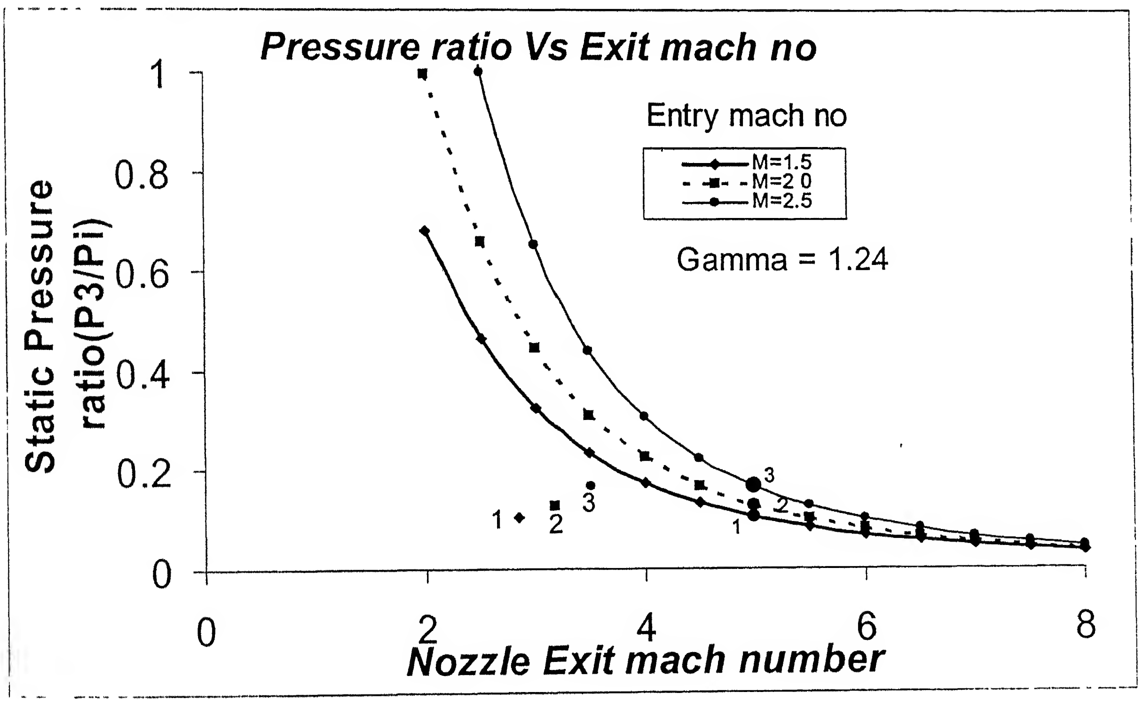


**Fig. 5.12 Effect of static pressure on nozzle length**

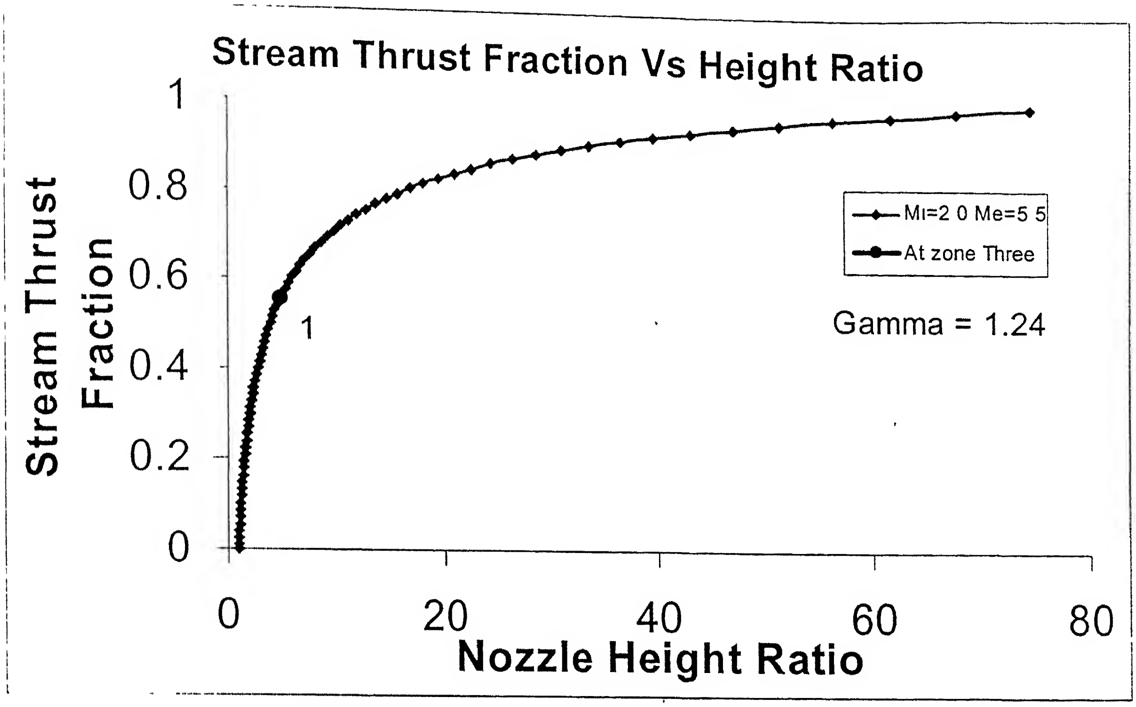


Fig. 5.13 Effect of Stream thrust function along the nozzle height

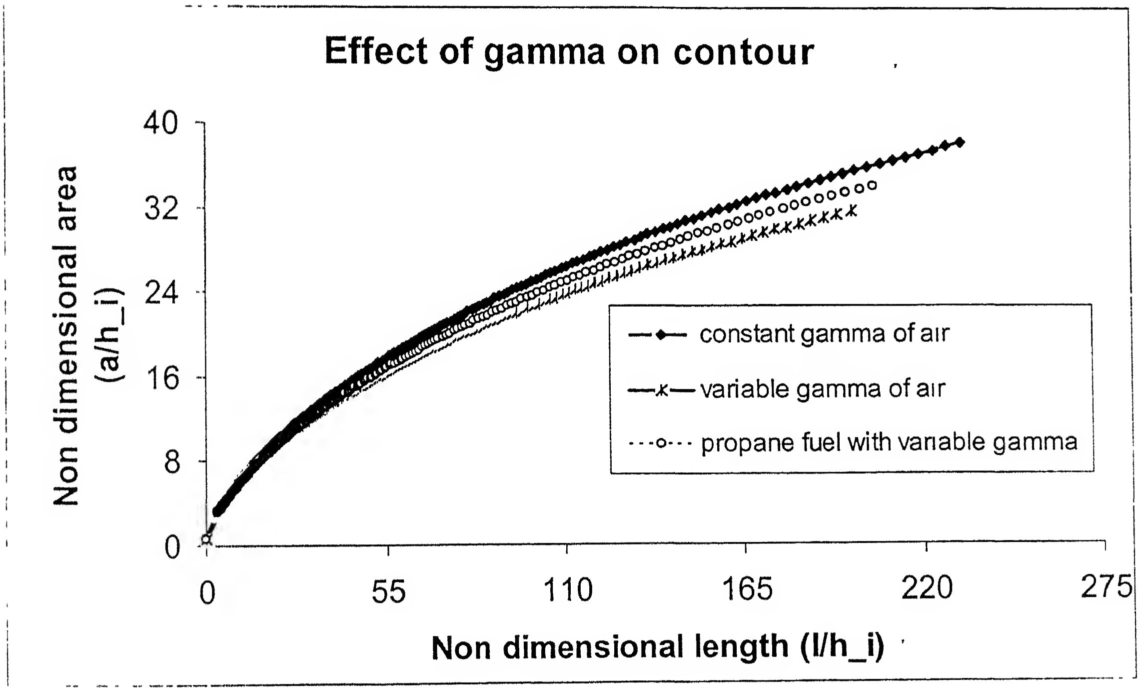


Fig. 5.14 Effect of gamma on nozzle contour

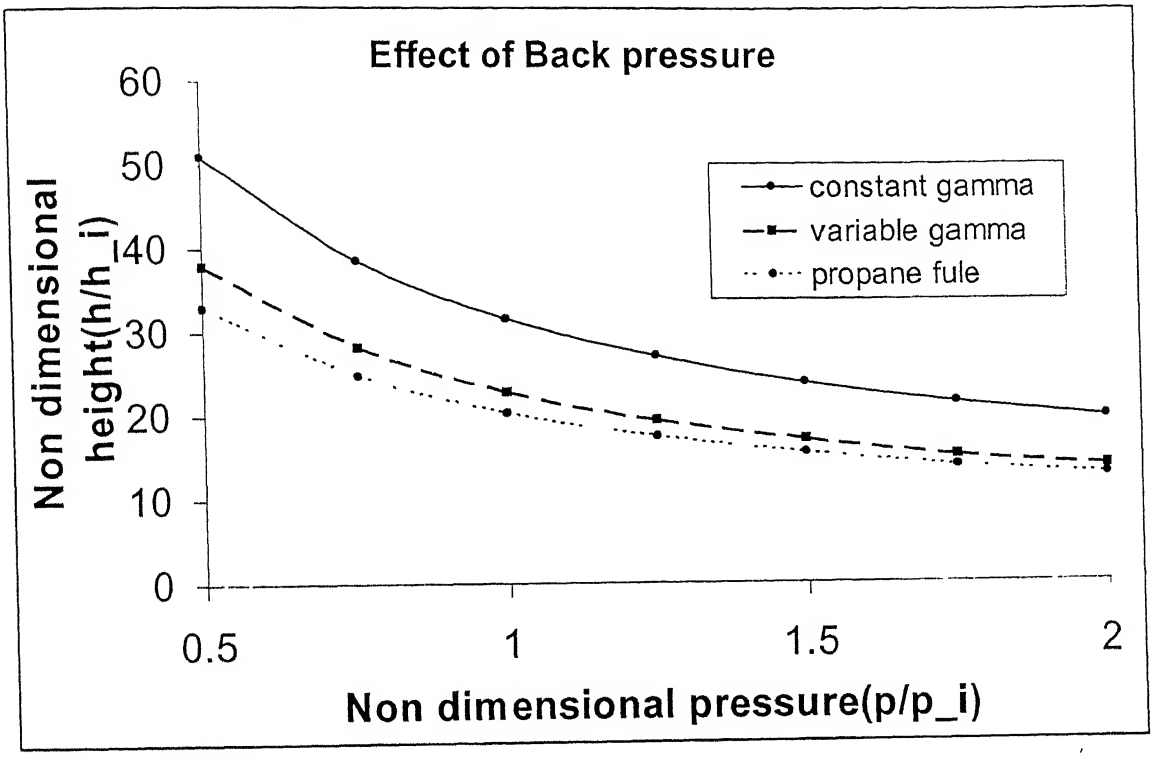


**Fig. 5.15 Effect of back pressure on exit area**

## Results for test case

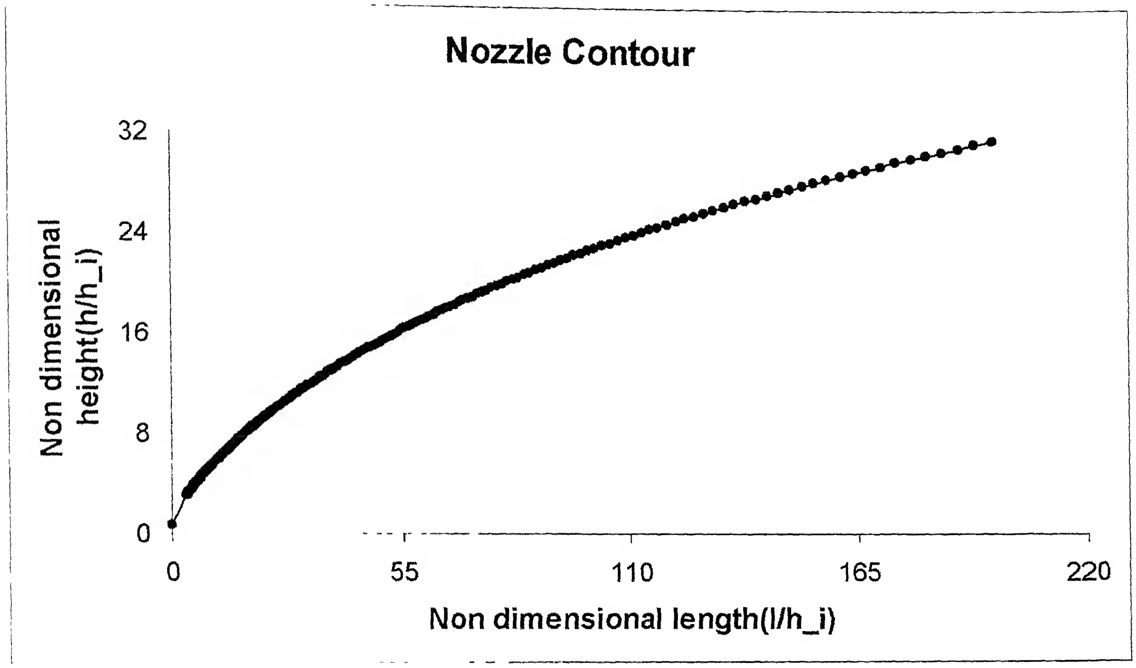


**Fig. 5.16 Nozzle contour**

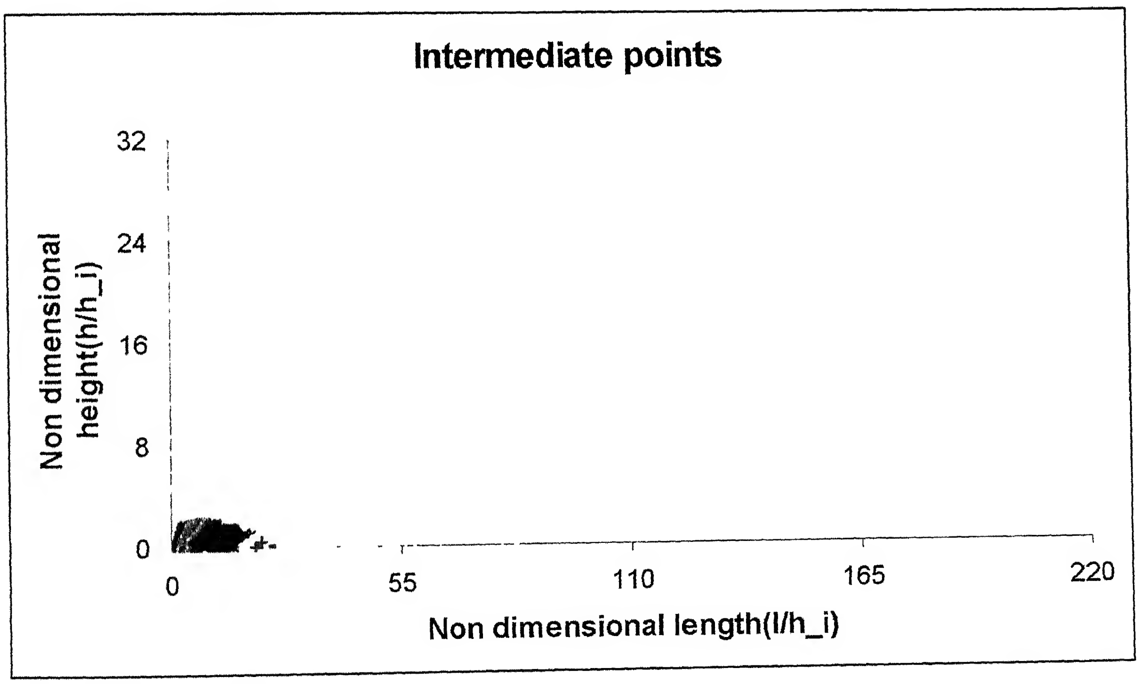


**Fig. 5.17 Intermediate points**

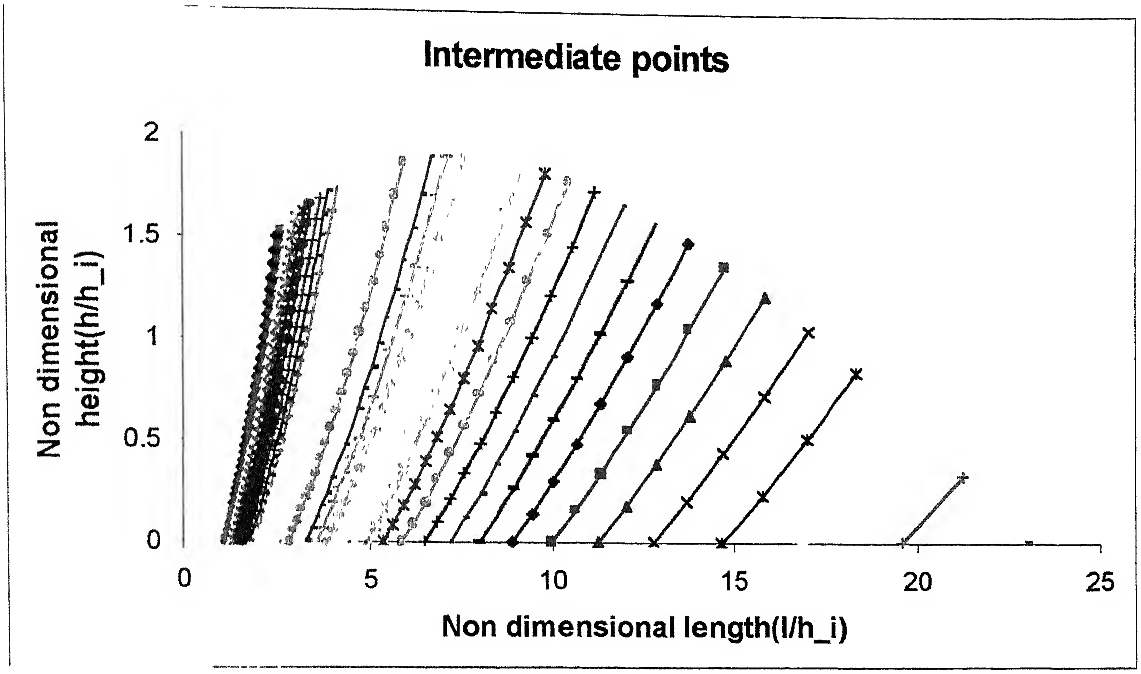


**Fig. 5.18 Intermediate points Enlarged view**

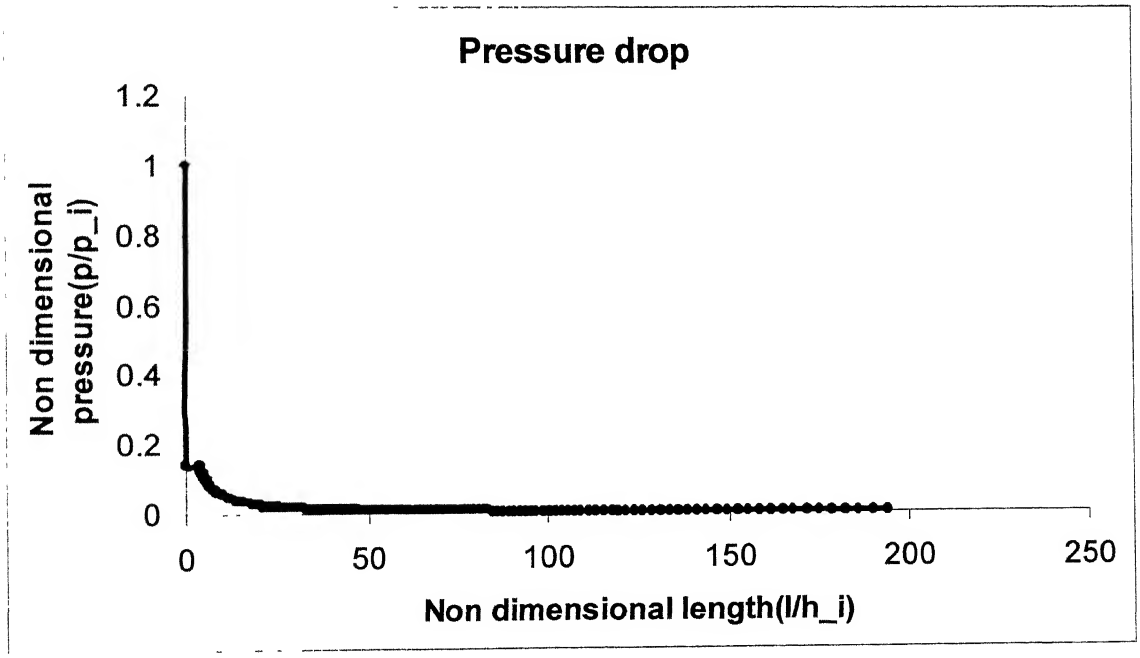


**Fig. 5.19 Pressure drop**

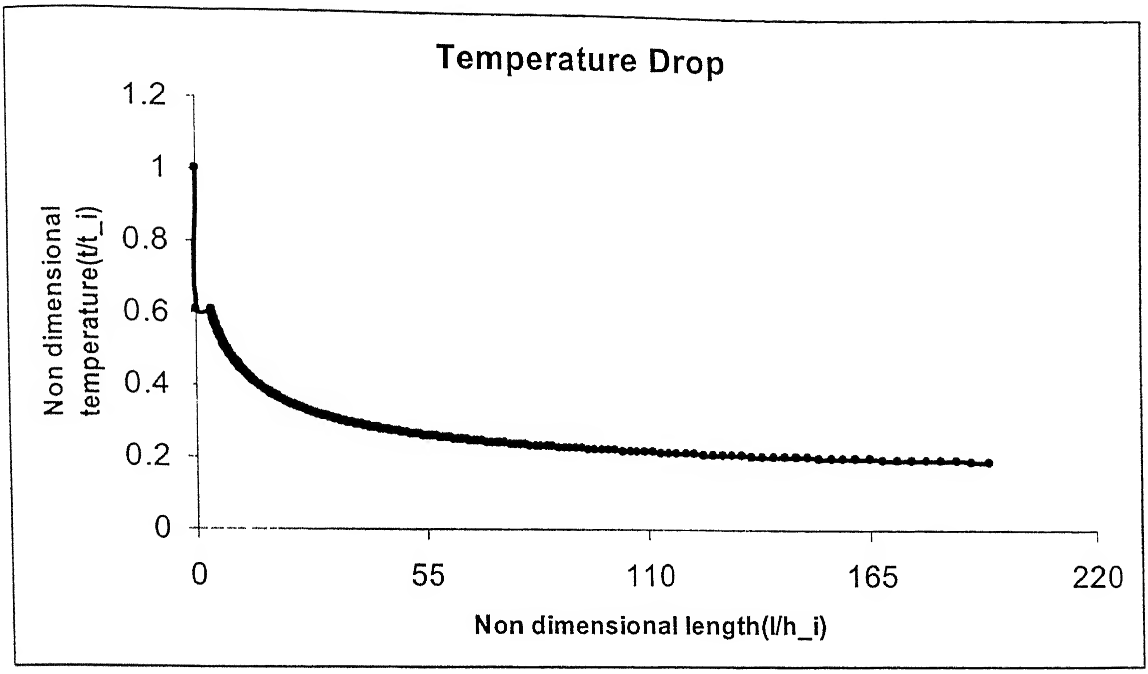


**Fig. 5.20 Temperature drop**

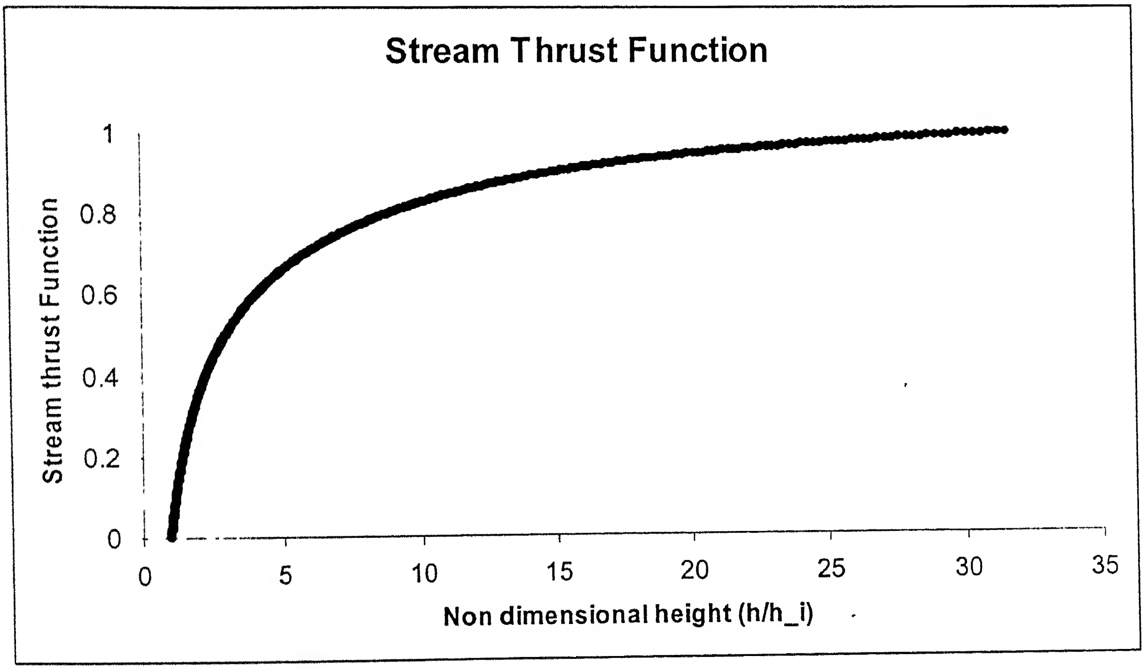


**Fig. 5.21 Variation of Stream thrust function**

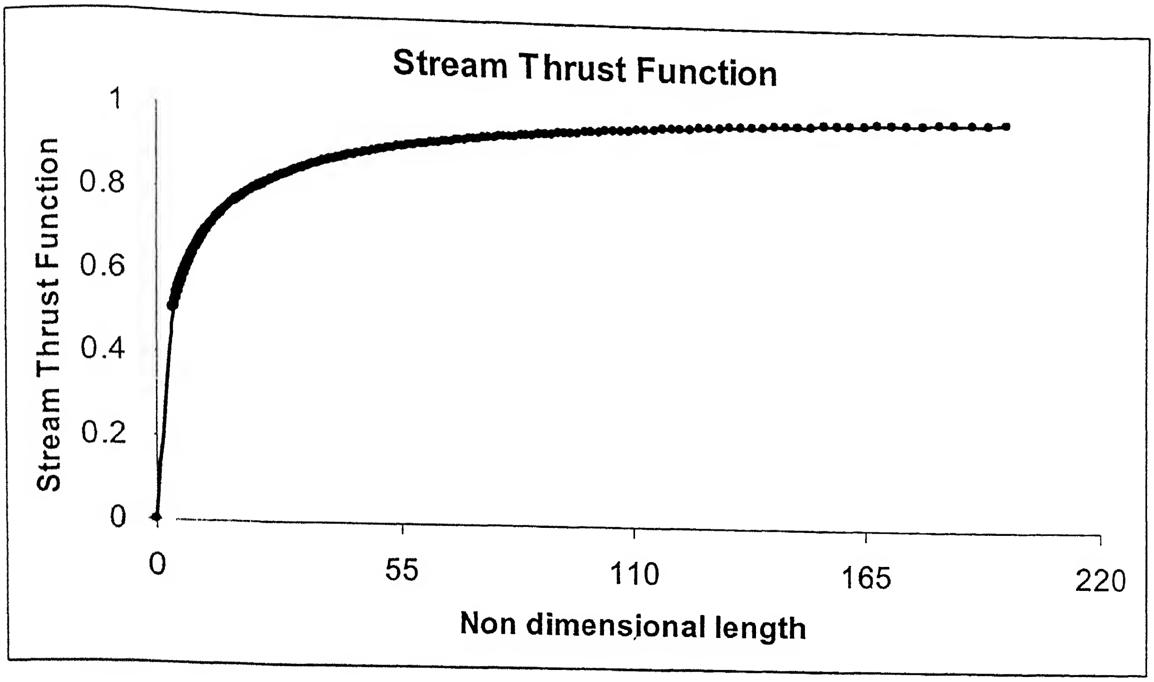


**Fig. 5.22 Stream thrust function along the length**

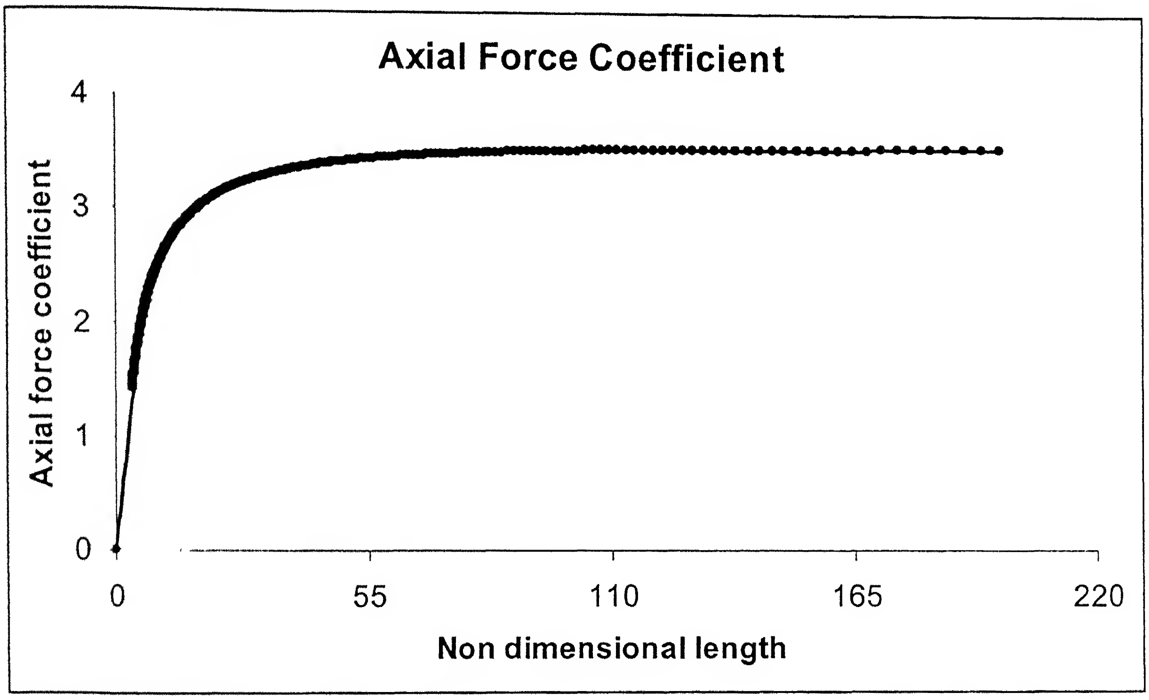


**Fig. 5.23 Axial force coefficient**

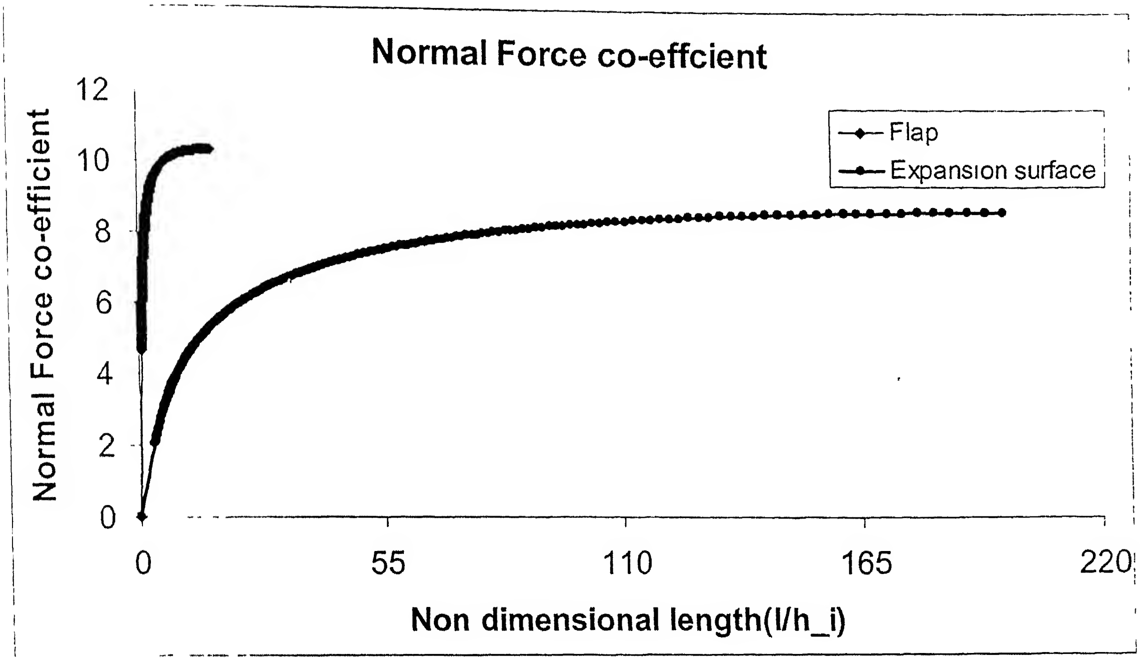


Fig. 5.24 Normal force coefficient

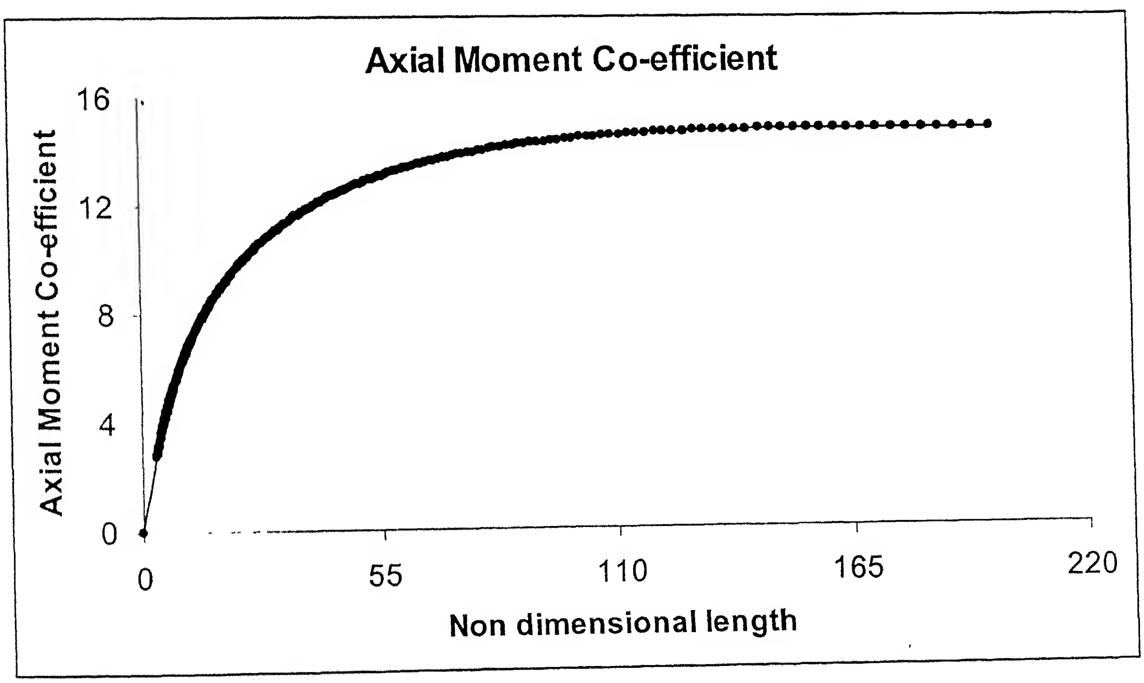


Fig. 5.25 Axial Moment coefficient

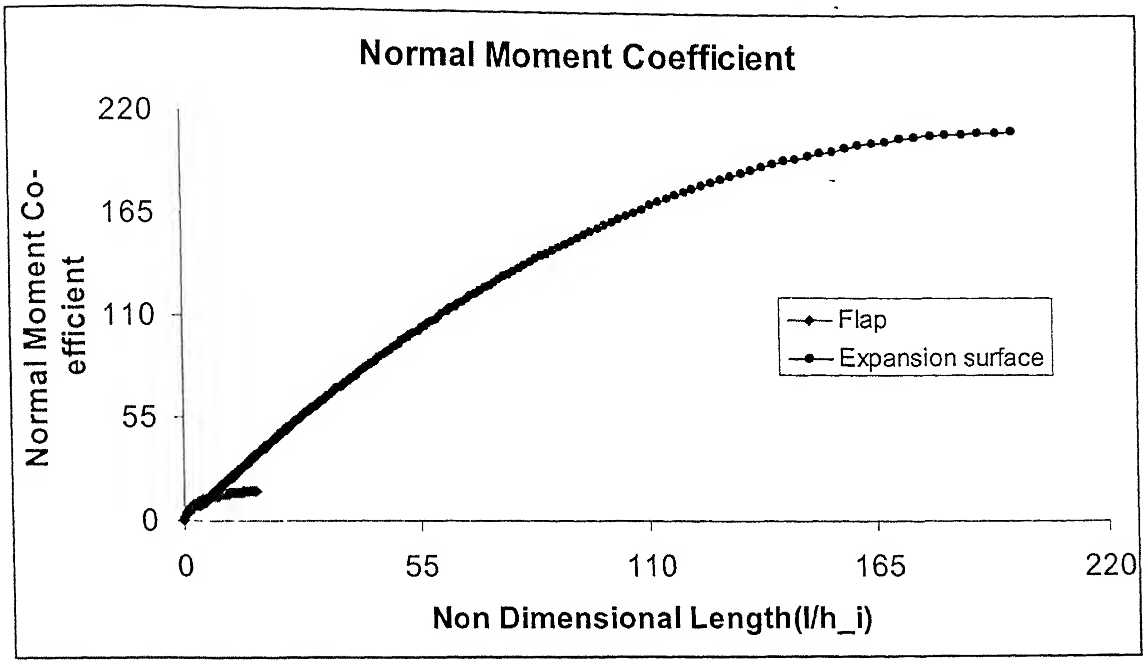


**Fig. 5.26 Normal Moment coefficient**

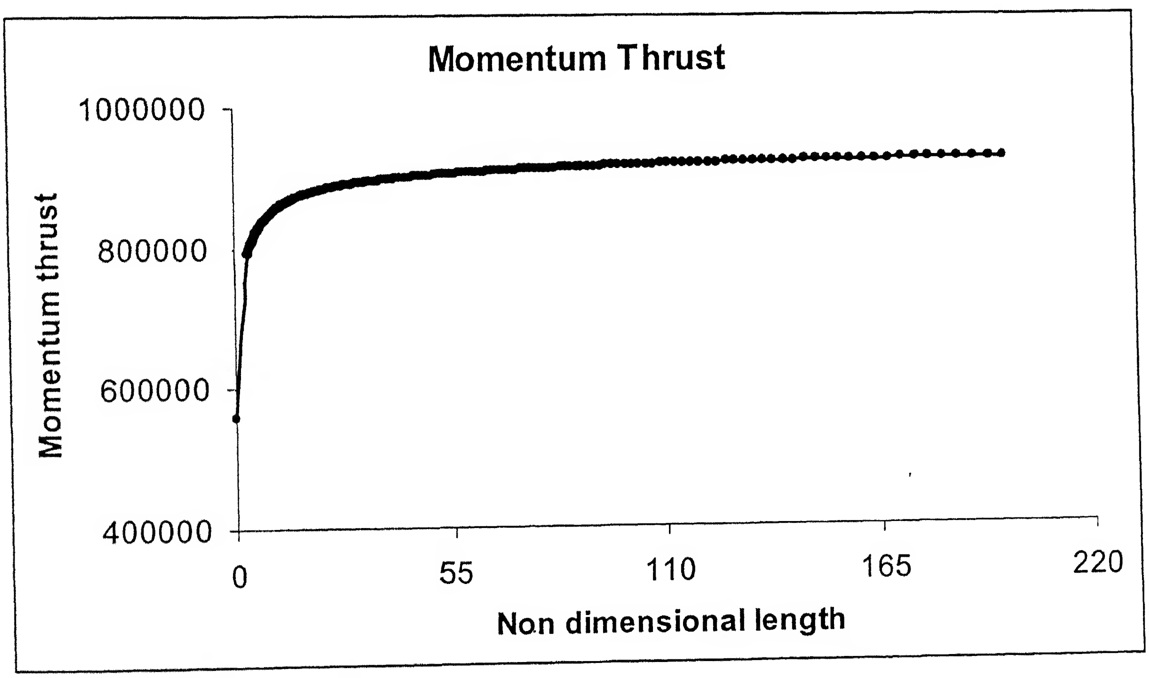


**Fig. 5.27 Momentum Thrust along the length**

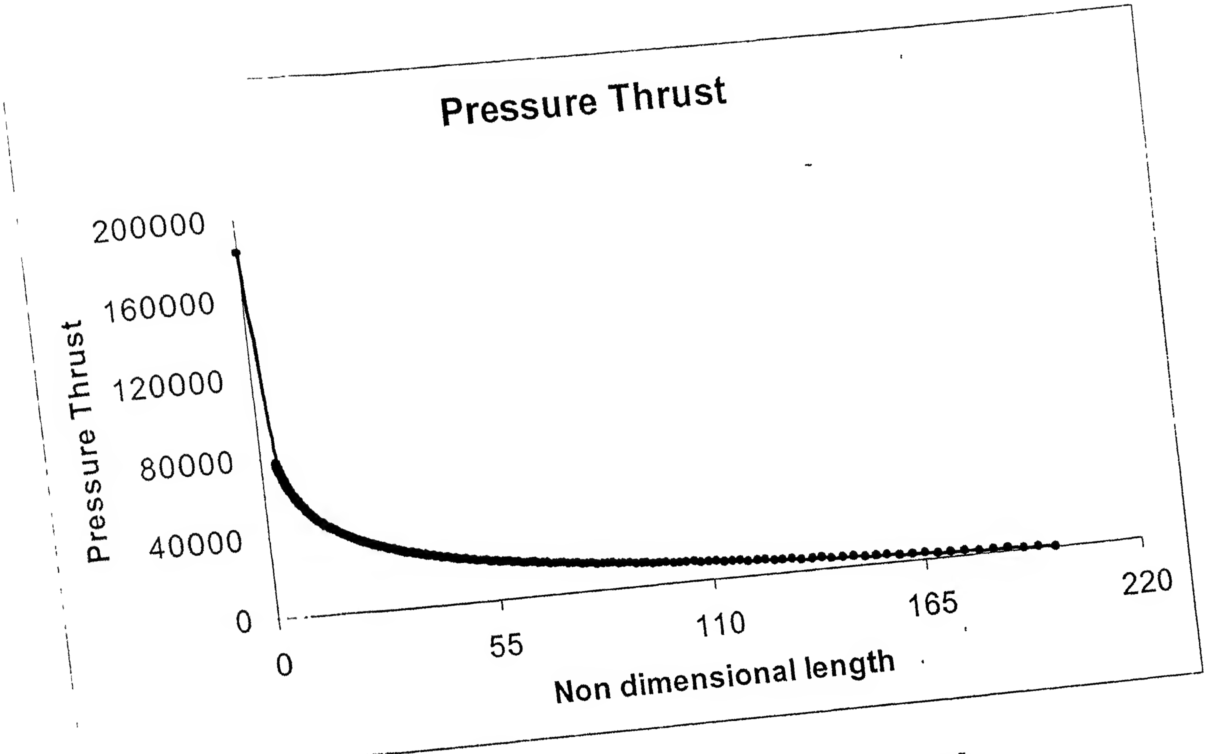


Fig. 5.28 Pressure thrust along the length

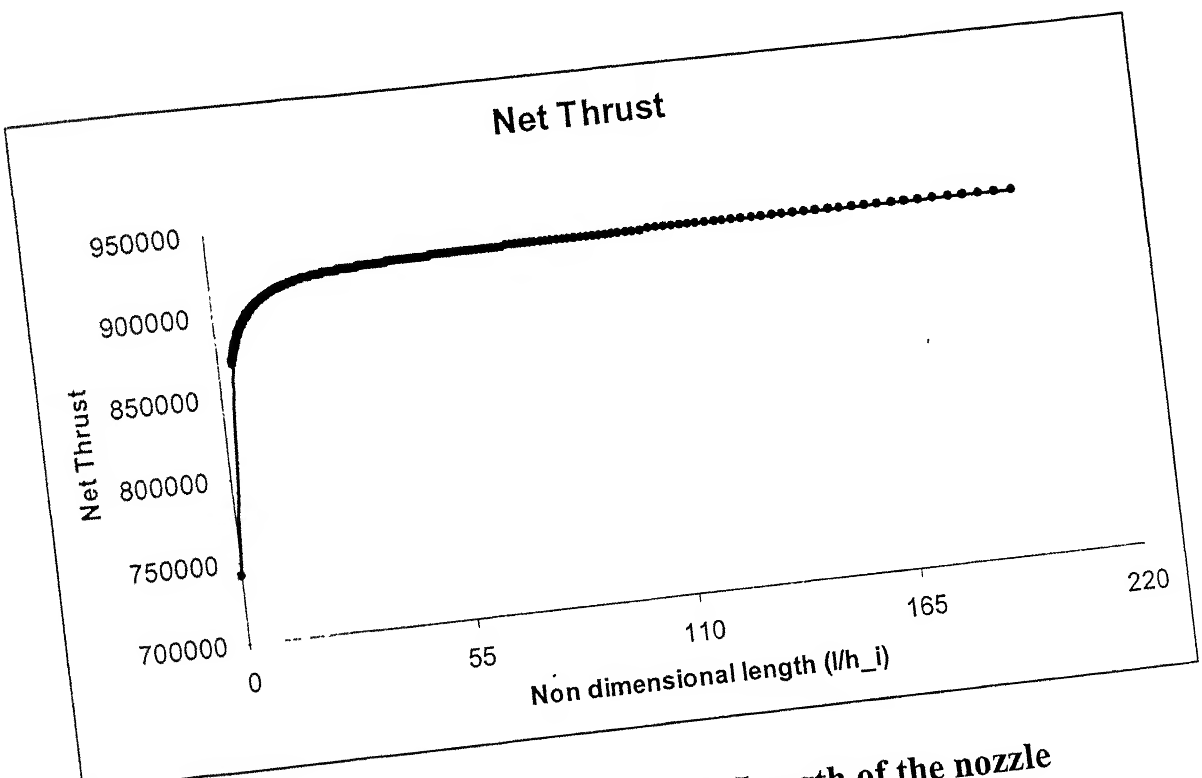


Fig. 5.29 Net Thrust along the Length of the nozzle

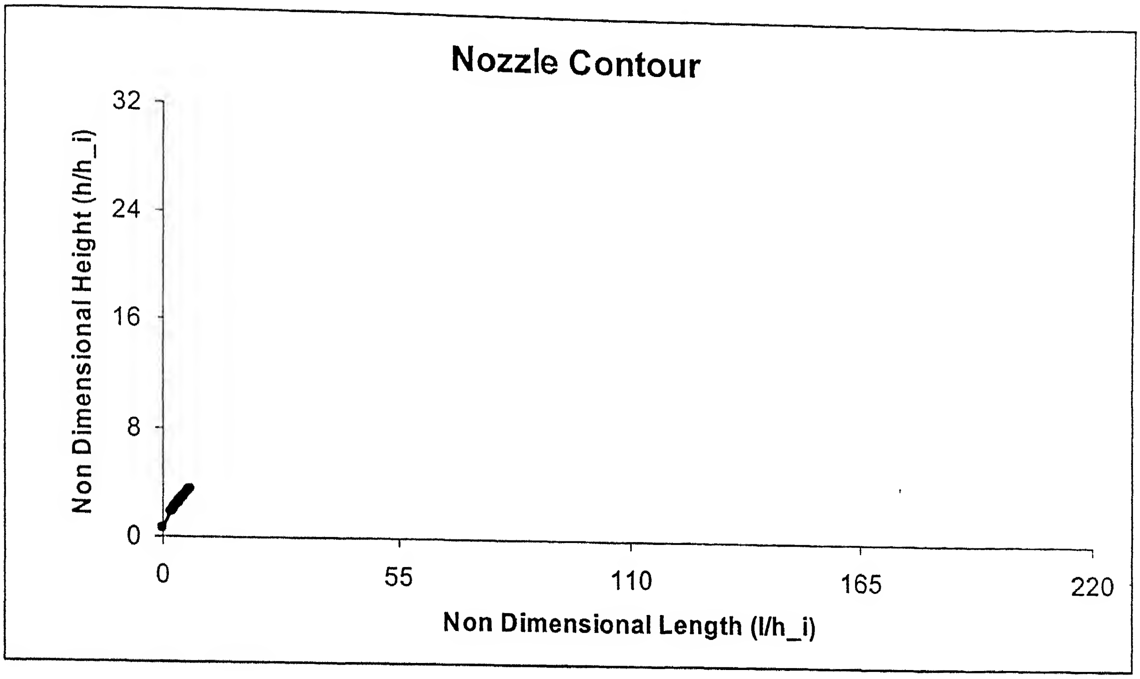


**Fig. 5.30 Truncated nozzle contour**

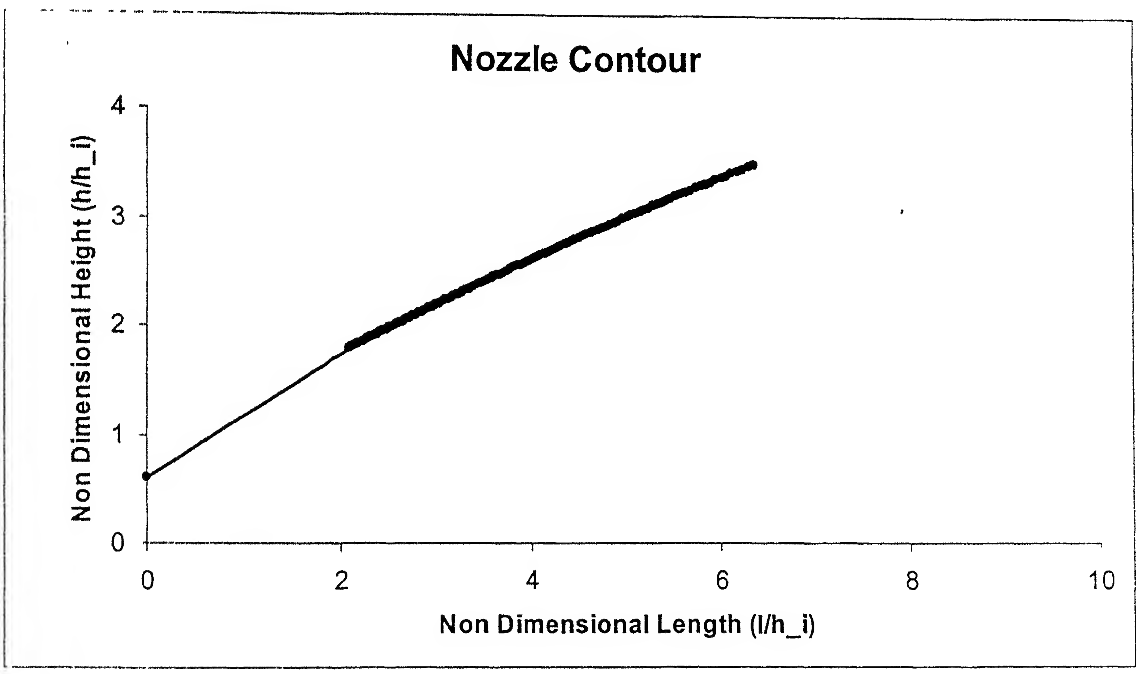


**Fig. 5.31 Truncated nozzle contour enlarged view**

# Results derived from Euler code

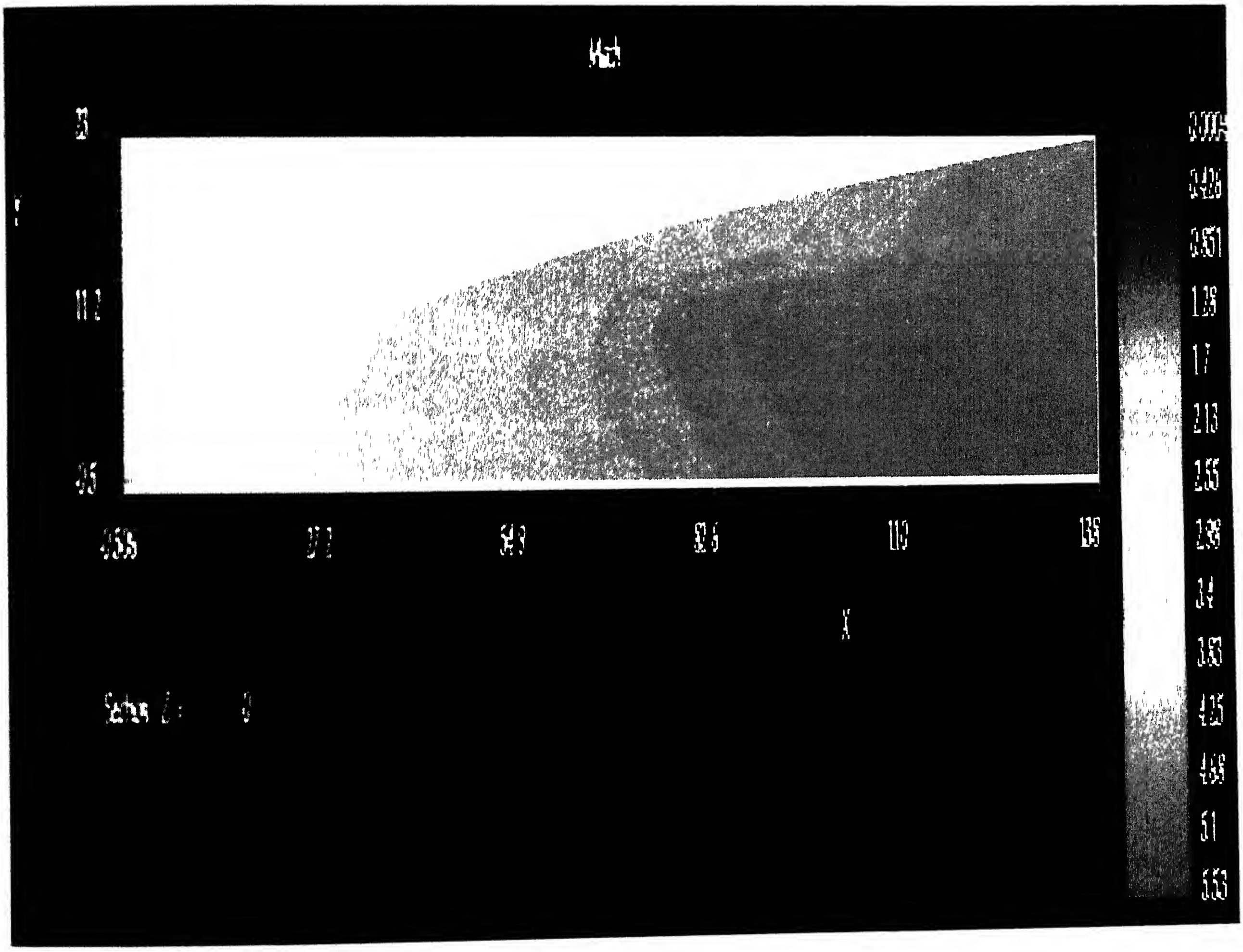


Fig. 5.32 Variation of Mach number along the length

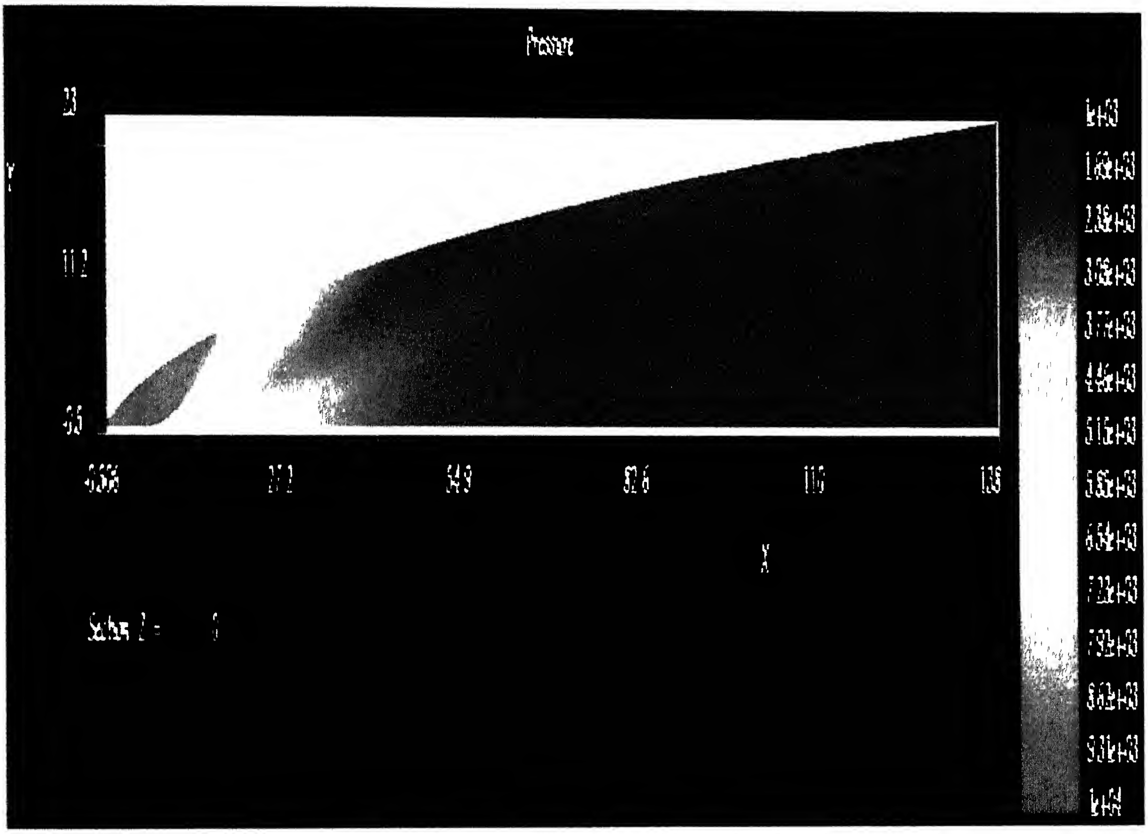

**Fig. 5.33 Variation of pressure along the length**

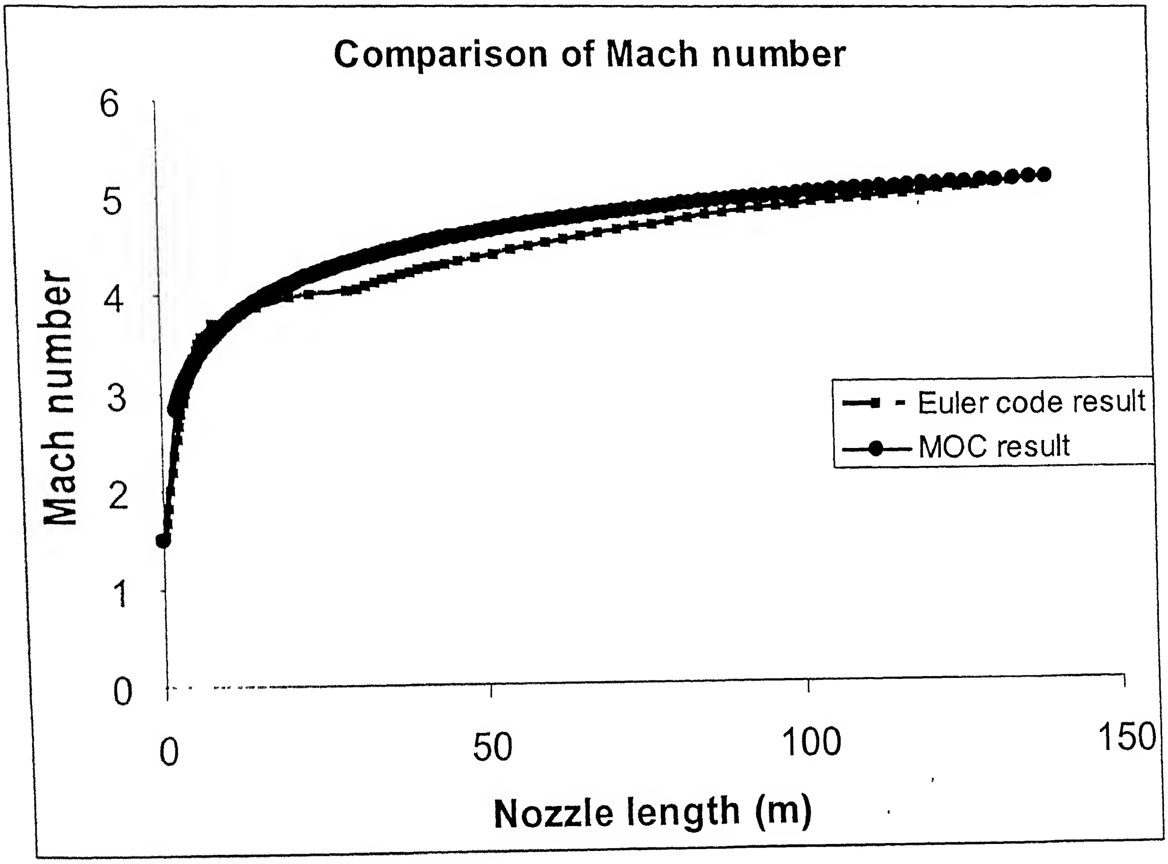


**Fig. 5.34 Comparison of Mach number along the wall**

# REFERENCE


1. Ahlberg, J. H., Hamilton, S., Migdal, D. and Nilson, E. N. "Truncated Perfect Nozzles in Optimum Nozzle Design", ARS Journal, 1961, pp. 614-620
2. Hoffman, J. D. "Design of Compressed Truncated Perfect Nozzles", AIAA Paper, 1995, pp. 85-172
3. Rao, G. V. R. "Exhaust Nozzle Contour for Maximum Thrust", Jet Propulsion, 1958, pp. 377-382
4. Rizkalla, O., Chinitz, W. and Erdos, J. I. "Calculated Chemical and Vibrational Non equilibrium Effects ın Hypersonic Nozzles", Journal of propulsion and power, 1990, pp. 50-57
5. Guha, A. "An Efficıent generic method for calculating the properties of combustion products", Journal of Power and Energy, 2000, pp. 375-387
6. Anderson, J. D. "Fundamentals of aerodynamics", McGraw-Hill Book company, 1985, pp. 450-461
7. Saad, M. A. "Compressible Fluid Flow", Prentice-Hall, 1985, pp. 345-466
8. RathaKrishnan, E., Gas Dynamics, Prentice-Hall of India, 2001, pp.273-296
9. Heiser, W. H. "Hypersonic Air breathing Propulsion" AIAA, 1994, pp. 387-423
10. Emanuel, G. "Gas Dynamics", Theory and Applications, AIAA, 1986, pp. 283-303
11. Hussaini, M. M. and Korte, J. J. "Investigation of Low-Reynolds-Number Rocket Nozzle Design Using PNS-Based Optimization Procedure", NASA Technical Memorandum 110295, 1996, pp. 1-12
12. Hill, P. G. and Peterson, C. R. "Mechanics and Thermodynamics of Propulsion", Addison-Wesley, 1992, pp. 141-269
13. Guha, A. "Performance of gas turbines with real gas effects", Journal of Power and Energy, 2001, pp. 375-387
14. Korte, J. J. "Invicid Design of Hypersonic Wind Tunnel for a Real gas", NASA Technical Memorandum AIAA-2000-0677, 2000, pp. 1-8

15. Sivells, J. C. "Aerodynamic Design of Axis.Symmetric Hypersonic Wind Tunnel Nozzles", Journal of Spacecraft and Rockets, 1970, pp. 1292-1299
16. Shapiro, A. H. "The dynamics and Thermodynamics of Compressible Fluid Flow", the Ronald Press Company, 1953
17. Beckwith, I. E., and Moore, J. A. "An Accurate and Rapid Method for the Design of Supersonic Nozzles", NASA TN 3322, 1995
18. Ruffin, S. M, and Spaid, F W. "Hypersonic Single Expansion Ramp Nozzle Simulations", Journal of Spacecraft and Rockets, 1992
19 Matesanz, A., Velazquez, A., and Rodriguez, M. "Aerodynamic Performance Prediction of Thrust-Vectoring Nozzles", Journal of Propulsion and Power, 1998, pp. 241-246
20. Keener, E. R., and Spaid, F. W. "Hypersonic Nozzle –After body Experiment: Flow Visualization and Boundary-Layer Measurements", Journal of Spacecraft and Rockets, 1996. pp. 326-337
21. Bradford, J. "Rapid Prediction of Aftbody Nozzle Performance in SCCREAM", AIAA 2002-3605, 2002, pp. 1-14
22. Haddad, A, and Moss, J. B. "Aerodynamic Design for Supersonic Nozzles of Arbitrary Cross Section", Journal of Propulsion and Power, 1990, pp. 740-746
23. Crown, J. C. "Supersonic Nozzle Design", NACA Technical note no. 1651, 1948, pp. 1-34
24. Wen-Lan Wang and Boyd, L D. "Continuum Breakdown in Hypersonic Viscous Flow", AIAA Paper 2002-0651, 2002, pp. 1-11
25. Anderson, J. D. "Modern Compressible Flow", McGraw-Hill, 1982, pp. 303-305
26. Curran, E. T, and Murthy, S. N. B. "Scramjet Propulsion", AIAA, Progress in Astronautics and Aeronautics, 2000
27. http://www.grc.nasa.gov/WWW/K-12/airplane/atmosmet.html